Computational Structural Mechanics and Dynamics

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Computational Mechanics

Assignment - 2.3 (Extra)

a)

$$A = A_i(1-\xi) + A_j(\xi) \qquad \xi = \frac{x - x_1}{l}$$

$$K^{(e)} = \int_0^1 EAB^TB\, l\, d\xi \quad ; B = \frac{1}{l}[-1 \quad 1]$$

$$= \int_0^1 \frac{EA}{l^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} l\, d\xi$$

$$= \frac{E}{l} \int_0^1 (A_i(1-\xi) + A_j\xi) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} d\xi$$

$$= \frac{E}{l} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \left(A_i\left(\xi - \frac{\xi^2}{2}\right) + A_j\frac{\xi^2}{2} \right)_0^1$$

$$= \frac{E}{l} \left[\frac{A_i}{2} + \frac{A_j}{2}\right] \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{E}{l} A \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \text{ where } A = \frac{A_i + A_j}{2}$$

b)

$$f^{(e)} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$q = \rho g A(\xi)$$

$$= \int_0^1 \rho g (A_i(1-\xi) + A_j(\xi)) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$= \rho g \int_0^1 \begin{bmatrix} A_i(1-\xi)^2 + A_j(\xi)(1-\xi) \\ A_i(1-\xi)(\xi) + A_j(\xi^2) \end{bmatrix} d\xi$$

$$= \rho g \begin{bmatrix} \left(A_i\left(\frac{\xi^3}{3} + \xi - \xi^2\right) + A_j\left(\frac{\xi^2}{2} - \frac{\xi^3}{3}\right)\right)_0^1 \\ \left(A_i\left(\frac{\xi^2}{2} - \frac{\xi^3}{3}\right) + A_j\left(\frac{\xi^3}{3}\right)\right)_0^1 \end{bmatrix}$$

$$= \rho g \begin{bmatrix} \frac{A_i}{3} + \frac{A_j}{6} \\ \frac{A_i}{6} + \frac{A_j}{3} \end{bmatrix} = \frac{\rho g}{6} \begin{bmatrix} 2A_i + A_j \\ A_i + 2A_j \end{bmatrix}$$

when $A_i = A_j$

$$f^{(e)} = \frac{\rho g}{6} \begin{bmatrix} 3A_i \\ 3A_i \end{bmatrix} = \frac{\rho g A_i}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{\rho g}{2} A \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$, where $A = A_i$

when $A_j = 0$

$$f^{(e)} = \frac{\rho g}{6} \begin{bmatrix} 2A_i \\ A_i \end{bmatrix} = \frac{\rho g A_i}{6} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{\rho g A}{6} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

where $A = A_i$

(c) $q = Q\,\delta(x-a)$

$\delta(x-a) = 0$ only when $x \neq a$

$$\int f(x)\,\delta(x-a) = f(a)$$

Q acts at $x = a$ from left end

$$\varepsilon = \frac{x - \cancel{x_1}^{\,0}}{l} \Rightarrow \varepsilon = \frac{x}{l} \Rightarrow \varepsilon - \frac{a}{l} = 0$$

$$f^{(e)} = \int_0^1 q \begin{bmatrix} 1-\varepsilon \\ \varepsilon \end{bmatrix} l\,d\varepsilon = \int Q\,\delta\left(\varepsilon - \frac{a}{l}\right) \begin{bmatrix} 1-\varepsilon \\ \varepsilon \end{bmatrix} l\,d\varepsilon$$

$$= Ql \int_0^1 \begin{bmatrix} (1-\varepsilon)\,\delta\left(\varepsilon - \frac{a}{l}\right) \\ \varepsilon\,\delta\left(\varepsilon - \frac{a}{l}\right) \end{bmatrix} d\varepsilon = Ql \begin{bmatrix} 1 - \frac{a}{l} \\ \frac{a}{l} \end{bmatrix} = Q \begin{bmatrix} l - a \\ a \end{bmatrix}$$

$$f^{(e)} = Q \begin{bmatrix} l - a \\ a \end{bmatrix}$$

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