4.1

$N_1^e(\xi) = a_0 + a_1\xi + a_2\xi^2 \quad N_2^e(\xi) = b_0 + b_1\xi + b_2\xi^2 \quad N_3^e(\xi) = c_0 + c_1\xi + c_2\xi^2$

$N_1(-1) = a_0 - a_1 + a_2 = -1 \rightarrow -a_1 - a_1 = 1 \rightarrow a_1 = -\frac{1}{2}$

$N_1(0) = a_0 = 0$

$N_1(1) = a_1 + a_2 = 0 \rightarrow -a_1 = a_2 \rightarrow a_2 = \frac{1}{2}$

$N_1^e(\xi) = -\frac{1}{2}\xi + \frac{1}{2}\xi^2 = \underline{\frac{1}{2}\xi(\xi - 1)}$

$N_2(-1) = b_0 - b_1 + b_2 = 0 \rightarrow b_1 = b_2$

$N_2(0) = b_0 = 0$

$N_2(1) = b_1 + b_2 = 1 \rightarrow 2b_1 = 1 \rightarrow b_1 = \frac{1}{2} = b_2$

$N_2^e(\xi) = \frac{1}{2}\xi + \frac{1}{2}\xi^2 = \underline{\frac{1}{2}\xi(1+\xi)}$

$N_3(-1) = c_0 - c_1 + c_2 = 0 \rightarrow c_2 = c_1 - 1 \rightarrow c_2 = -1$

$N_3(0) = c_0 = 1$

$N_3(1) = c_0 + c_1 + c_2 = 0 \rightarrow 1 + c_1 + c_2 = 0 \rightarrow 1 + c_1 + c_1 - 1 \rightarrow c_1 = 0$

$N_3(\xi) = \underline{1 - \xi^2}$

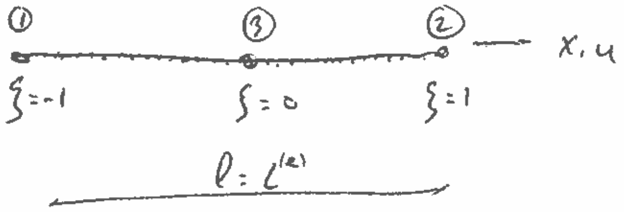
$N_1^e(\xi) + N_2^e(\xi) + N_3^e(\xi) = \frac{1}{2}\xi(\xi-1) + \frac{1}{2}\xi(1-\xi) + 1 - \xi^2 = \xi^2 + 1 - \xi^2 = \underline{\underline{1}}$

4.2

$$\begin{bmatrix}1\\x\\u\end{bmatrix} = \begin{bmatrix}1&1&1\\x_1&x_2&x_3\\u_1&u_2&u_3\end{bmatrix}\begin{bmatrix}N_1^e\\N_2^e\\N_3^e\end{bmatrix}$$

$x_1 = 0,\ x_2 = \ell,\ x_3 = (\frac{1}{2}+\alpha)\ell$ $\qquad \ell = x_1 - x_2$

$\{-\frac{1}{2} < \alpha < \frac{1}{2}\}$



1.- $J = \frac{dx}{d\xi}$?

$$\left\{\begin{array}{l} 1 = N_1^e + N_2^e + N_3^e \\ x = x_1 N_1^e + x_2 N_2^e + x^3 N_3^e \\ u = u_1 N_1^e + u N_2^e + u_3 N_3^e \end{array}\right\} \to \left.\begin{array}{l} x = \ell N_2^e + (\frac{1}{2}+\alpha)\ell N_3^e \quad (x_1 = 0) \\ N_2^e = \frac{1}{2}\xi(\xi+1) \\ N_3^e = 1-\xi^2 \end{array}\right\} \to x = \frac{\ell}{2}\xi(\xi+1) + (\frac{1}{2}+\alpha)\ell(1-\xi$$

$$\frac{dx}{d\xi} = \ell\xi + \frac{\ell}{2} + (\frac{1}{2}+\alpha)\ell(-2\xi) = \ell\xi + \frac{\ell}{2} - \xi\ell - 2\alpha\ell\xi = \ell(\frac{1}{2} - 2\alpha\xi) \Rightarrow \underline{\underline{\ell(\frac{1}{2} - 2\alpha\xi)}}$$

a)

$$\left.\begin{array}{l} \alpha = -\frac{1}{4} \to J = (\frac{1}{2} + \frac{1}{2}\xi) \\ \alpha = \frac{1}{4} \to J = \ell(\frac{1}{2} - \frac{1}{2}\xi) \end{array}\right\} J \geq 0 \ \forall \ (-1 \leq \xi \leq 1)$$

b) $\alpha = 0 \to J = \frac{\ell}{2}$ simpler etc.

2.- $e = \frac{du}{dx} = B u^e \to B = \frac{dN}{dx} = J^{-1}\frac{dN}{d\xi}$

$$\left.\begin{array}{l} N_1 = \frac{1}{2}\xi(\xi-1) \to \frac{dN_1}{d\xi} = \xi - \frac{1}{2} \\ N_2 = \frac{1}{2}\xi(\xi+1) \to \frac{dN_2}{d\xi} = \xi + \frac{1}{2} \\ N_3 = 1-\xi^2 \to \frac{dN_3}{d\xi} = -2\xi \end{array}\right\} J = (\frac{1}{2} - 2\alpha\xi)\ell \to J^{-1} = \frac{1}{\ell(\frac{1}{2} - 2\alpha\xi)}$$

$$B = J^{-1}\frac{dN}{d\xi} = \frac{1}{\ell(\frac{1}{2}-2\alpha\xi)}\begin{bmatrix}\xi - \frac{1}{2}\\ \xi + \frac{1}{2}\\ -2\xi\end{bmatrix} = \begin{bmatrix}\frac{(\xi-\frac{1}{2})}{\ell(\frac{1}{2}-2\alpha\xi)}\\ \frac{(\xi+\frac{1}{2})}{\ell(\frac{1}{2}-2\alpha\xi)}\\ -\frac{2\xi}{\ell(\frac{1}{2}-2\alpha\xi)}\end{bmatrix}$$

3.-

$$K^e = \int_0^l EA\, B^T B\, dx = \int_{-1}^{1} EA\, B^T B\, J\, d\xi \quad dx$$

$$B = \left[\frac{\partial N_1}{\partial x}, \frac{\partial N_2}{\partial x}, \frac{\partial N_3}{\partial x} \right] = J^{-1} \left[\frac{\partial N_1}{\partial \xi}, \frac{\partial N_2}{\partial \xi}, \frac{\partial N_3}{\partial \xi} \right]$$

$$J = \frac{dx}{d\xi} \;\text{--}\; dx = J\, d\xi \qquad \frac{dN}{dx} = \frac{dN}{d\xi} \cdot \frac{d\xi}{dx} = \frac{dN}{d\xi} J^{-1}$$

$$K = \int_0^l EA\, B^T B\, dx = \int_{-1}^{1} EA\, B^T B\, J\, d\xi = \int_0^1 EA\, B^T B\, J\, d\xi$$

4.3

$$N_5(\xi,\eta) = c\,L_{1-2}\,L_{4-3}\,L_{4-1}\,L_{3-2} \qquad \begin{cases} L_{1-2} = (\eta+1) \\ L_{4-3} = (\eta-1) \\ L_{4-1} = (\xi+1) \\ L_{3-2} = (\xi-1) \end{cases}$$

$$N_5(\xi,\eta) = c(\eta^2-1)(\xi^2-1)$$

$$N_5(0,0) = c = 1$$

$$N_4 = \underline{N_4} + \alpha N_5$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta) + \alpha(\eta^2-1)(\xi^2-1) \quad \text{en} \quad N_4(0,0) \longrightarrow \alpha = -\frac{1}{4}$$

Funciones de forma $N_i = \underline{N_i} + \alpha N_5 \Rightarrow N_i = N_i - \frac{1}{4}N_5$

$$\begin{cases} N_1^e = \frac{1}{4}(1-\xi)(1-\eta) - \frac{1}{4}(\eta^2-1)(\xi^2-1) \\ N_2^e = \frac{1}{4}(1+\xi)(1-\eta) - \frac{1}{4}(\eta^2-1)(\xi^2-1) \\ N_3^e = \frac{1}{4}(1+\xi)(1+\eta) - \frac{1}{4}(\eta^2-1)(\xi^2-1) \\ N_4^e = \frac{1}{4}(1-\xi)(1+\eta) - \frac{1}{4}(\eta^2-1)(\xi^2-1) \\ N_5^e = (\eta^2-1)(\xi^2-1) \end{cases}$$

$$N_1^e + N_2^e + N_3^e + N_4^e + N_5^e = (\eta^2-1)(\xi^2-1) - (\eta^2-1)(\xi^2-1) + \frac{1}{4}\Big[(1-\xi-\eta+\eta\xi)$$
$$+ (1-\eta+\xi-\eta\xi) + (1+\eta+\xi+\eta\xi) + (1-\xi+\eta-\eta\xi) = \frac{4}{4} = 1$$