



Assignment 7- Plates

Assignment a: Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate. Using 5x5 Mesh

$t=0,001$; $t=0,010$; $t=0,020$; $t=0,100$; $t=0,400$.

Kirchoff's plate theory can only be used for representations of thin plates with an average thickness / side thickness < 0.1 . Instead using the formulation of Reissner and Mindlin we can obtain effects of deformation transversal to the cut lastly large. However, some difficulties arise when Reissner-Mindlin elements are used for sheet metal situations due to the excessive influence of the transverse shear deformation terms. This defect is comparable to that already seen when the elements of the Timoshenko beam are applied in thin rays. Le length is about $L = 1 \text{ m}$ where I applied a uniform load $Q = 1 \frac{N}{m}$ and simple support in each side of the square:

$$\begin{cases} x \text{ direction: } w = \theta_x = 0 \\ y \text{ direction: } w = \theta_y = 0 \end{cases}$$

We can compare the MZC method with the Reissner-Mindlin method by the maximum displacements that we obtain for each thickness.

Thickness	Max displacement RM	Max displacement MZC	$\Delta - \text{maxdisplacement}$
0,001	3641690	3860480	$3860480 - 3641690 = 218790$
0,01	3642,77	3860,48	$3860,48 - 3642,77 = 217,71$
0,02	482,56	482,56	0
0,1	3,85	3,86	0,01
0,4	0,11	0,06	-0,05

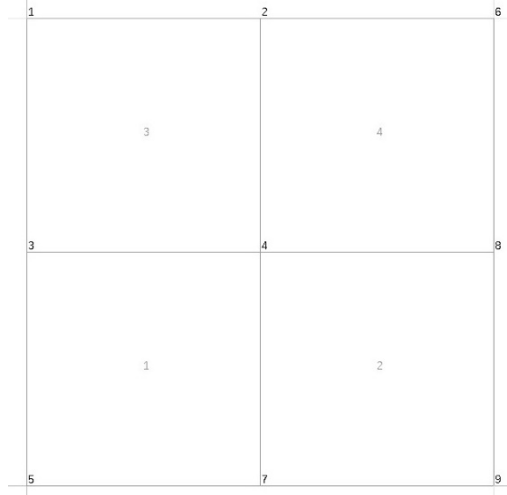
We can observe that for the first two thicknesses analyzed, the difference between MZC and Reissner-Mindlin is very large, this due to the shear locking defect mentioned before. In fact the Reissner-Mindlin's elements seems to be stiffer than the reality. We can also observed that for $t > 0,1$ the two solution are quite similar each other. Therefore, as stated by the theory, confirmed the affirmation of the whole thickness of the slabs, the theory of the MZC elements is more suitable, because the shear locking defect for RM solutions is reduced.

Assignment b: Define and verify a patch test mesh for the MCZ element. Discuss the observed results.

The patch test is an indicator of the quality of a mesh. Refinement of the mesh is known; numerical solutions can be obtained that are increasingly convergent to the exact solution. If this happens, the test has been verified.

In this example the Patch Test was performed for the formulation of the MZC element, with a mesh of 2X2 as shown in the figure for a plate with a thickness $t = 0.1\text{m}$.

The equations with which the displacement will be evaluated for each node are the following:



$$z_1 = -0,01x - 0,01y;$$

$$z_2 = -\frac{0,001x^2 + 0,001xy + 0,001y^2}{2}$$

nodes	1	2	3	4	5	6	7	8	9
X	0	0,5	0	0,5	0	1	0,5	1	1
Y	1	1	0,5	0,5	0	1	0	0,5	0
z₁	-0,01	-0,015	-0,005	-0,01	0	-0,02	-0,005	-0,015	-0,01
z₂	-0,0005	-0,00875	-0,000125	-0,0003750	0	-0,0015	-0,000125	-0,000875	-0,0005

The results are not so differ each other. However, using the second equation gives a greater error due to the boundary conditions, which have quadratic functions. The use of a quadratic or refined mesh will solve this problem.