Computational Structural Mechanics and Dynamics
Assignment 7

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1 BEAMS

Assignment 7.1

In order to compute Timoshenko 2 nodes Beam element with reduce integration we have to use the following shear stiffness matrix,

$$\mathbf{K}_s^e = (\frac{GA^*}{l})^e \begin{bmatrix} 1 & \frac{l^e}{2} & -1 & \frac{l^e}{2} \\ \frac{l^e}{2} & \frac{(l^e)^2}{4} & -\frac{l^e}{2} & \frac{(l^e)^2}{4} \\ -1 & -\frac{l^e}{2} & 1 & -\frac{l^e}{2} \\ \frac{l^e}{2} & \frac{(l^e)^2}{4} & -\frac{l^e}{2} & \frac{(l^e)^2}{4} \end{bmatrix}$$

Where, (G) is Shear modulus, 'A' is cross sectional area of the beam and (l) is the length of the beam.The modified matlab code for both reduced and reduced integration can be accessed in zip file namely '**Timored**',where the modified $\mathbf{K}_s^e$ and stress can be seen.

Assignment 7.2

In this assignment we are analysing the cross section of the Simply Supported beam that has been given, with the methods like Euler-Bernouli, Timoshenko full integration and Timoshenko reduced integration that is 1 point gauss integration.And the plot of (a/l) versus maximum displacement(w),Moments and Shear force for three elements is analysed and compared for the given data: E=21000,$\nu = 0.25$,Uniformly distributed load,P=1.0 N/m and $l = 4$.

For Simply Supported beam,theoretical values for deflection/displacement,Moment and Shear Force are calculated using the formula below:

Euler-Bernoulli:
Deflection/Displacemnt(max),

$$W_{act} = \frac{5Pl^4}{384EI} \quad m$$

Where for sqaure c-s beam,

$$I = \frac{a^4}{12}$$

Moment,

$$M_{act} = \frac{Pl^2}{8} = 2Nm$$

Timoshenko Integration(Reduced and Full):
Deflection/Displacemnt(max),

$$W_{act} = [\frac{5Pl^4}{384EI} + \frac{I}{GA^*}] \quad m$$

Where for rectangular beam,

$$A^* = \frac{5}{6} \times A$$

Moment,

$$M_{act} = \frac{Pl^2}{8} = 2Nm$$

Shear Force,

$$Q_{act} = \frac{Pl}{2} = 2N$$

We have to generate the 'input file' which can read as a matlab code in my zip file named '`simplysup_64.m`', which is input file for all the three methods(Euler-Bernoulli,Timoshenko Full Integration and Reduced Integration).The zip files attached contains the complete matlab codes for 3 methods namely **Euler**,**Timofull**,**Timored**,which has to be run separately to obtain results.The results obtained are explained below.

I have calculated numerical values of maximum deflection, moment and shear force for different values of cross sectional length (a), i.e for a=0.001, 0.005, 0.010, 0.020, 0.050, 0.100, 0.200 and 0.4 as asked in question 2 and are plotted against (a/l) and are analysed as shown below,considering for 64 element mesh as given in question.

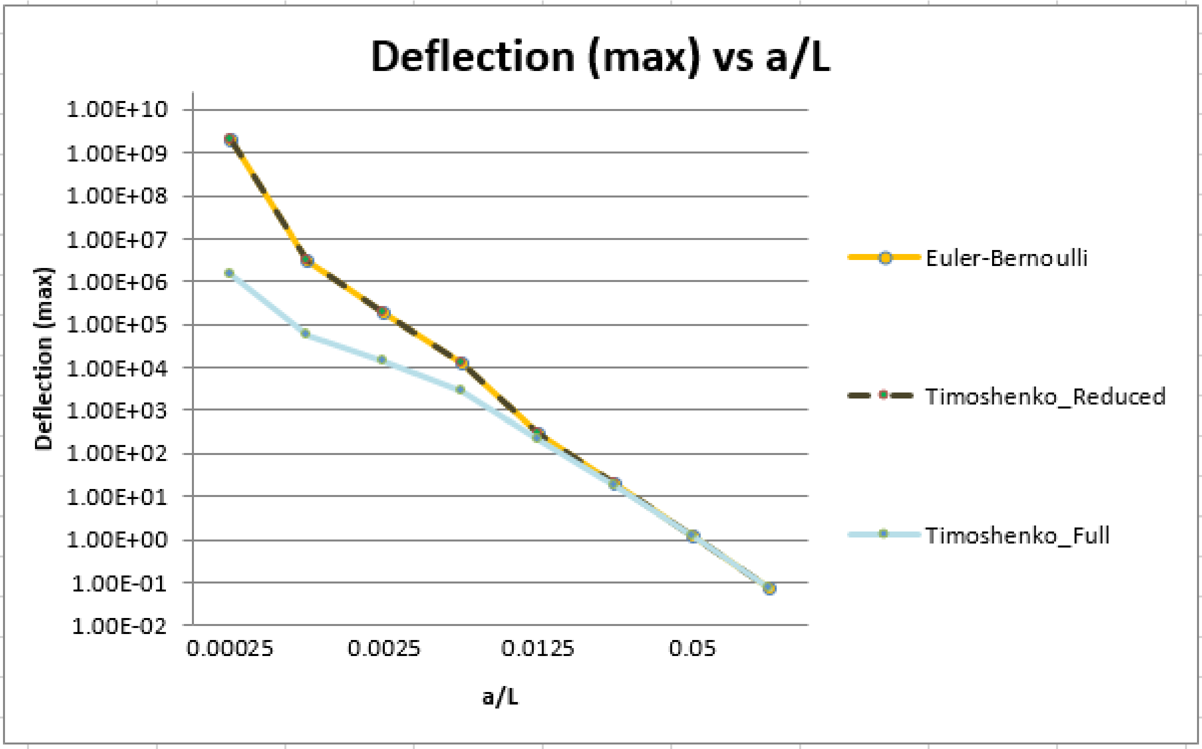

Figure 1: Plot of Max.Deflection versus (a/l)

From the above plot we can conclude that Timoshenko reduced integration is same as that of the Euler-Bernoulli that is they both behave the same, not exactly but very close to each other. As and when the ratio of (a/l)is increased the maximum negative deflection converges to zero that is it decreases very close to zero.But in case Timoshenko full integration,for lower (a/l) i.e, for thinner beam the max.displacements are not matching with the other two methods,but after increasing (a/l) or thickness of beam it is converging with the exact solutions.And hence we can conclude that our analysis is correct as per the theory.

Below plot shows the variation of Moment versus ratio of (a/l),

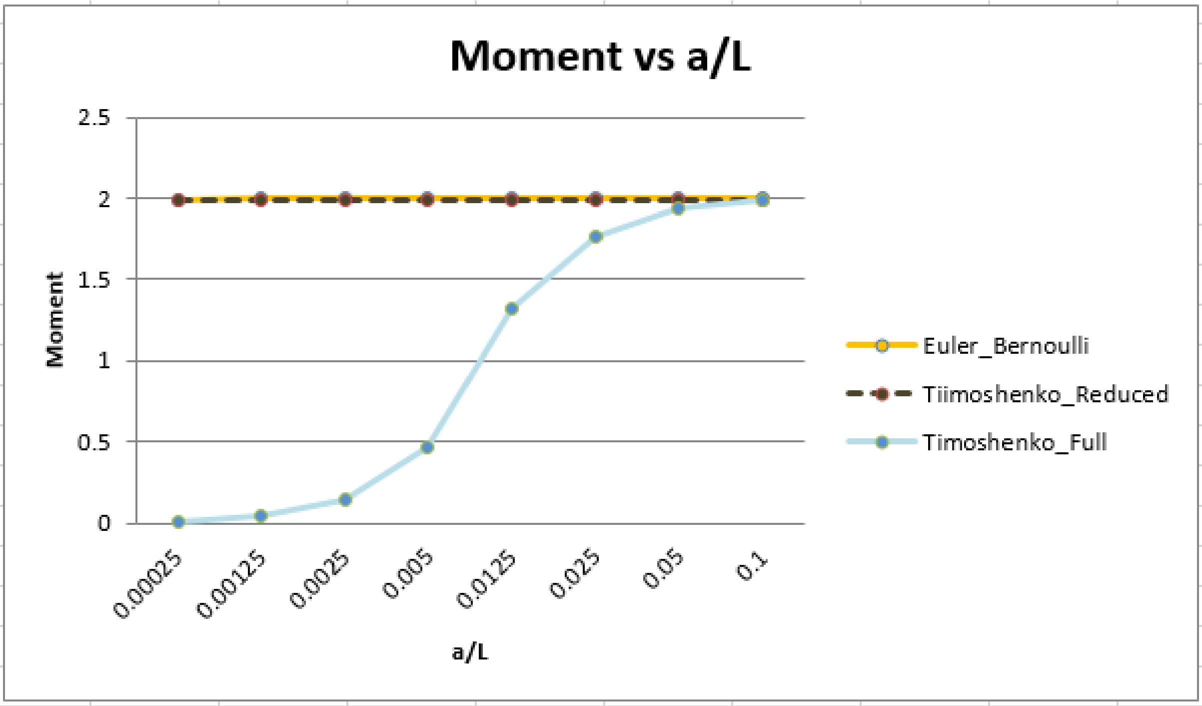

Figure 2: Plot of Moment versus (a/l)

When the simulation of moments were analysed,for which the analytical result is '$2Nm$' and when the analytical result was compared with the numerical solution,we found that Timoshenko reduced integration and Euler-Bernoulli moment curves are very close to analytical solution and also both methods behave same in the analysis of the moment as well.As the cross sectional length(a) was increased,numerical solution is very close to the analytical result.But while in other hand Timoshenko full integration method behaves abruptly for the lower (a/l).But when the ratio of (a/l) was increased,even this method is close to the analytical solution,which can be observed from Figure 2 above. Thus we can say that Timoshenko reduced integration and Euler-Bernoulli methods are best suited for integration of beam element.

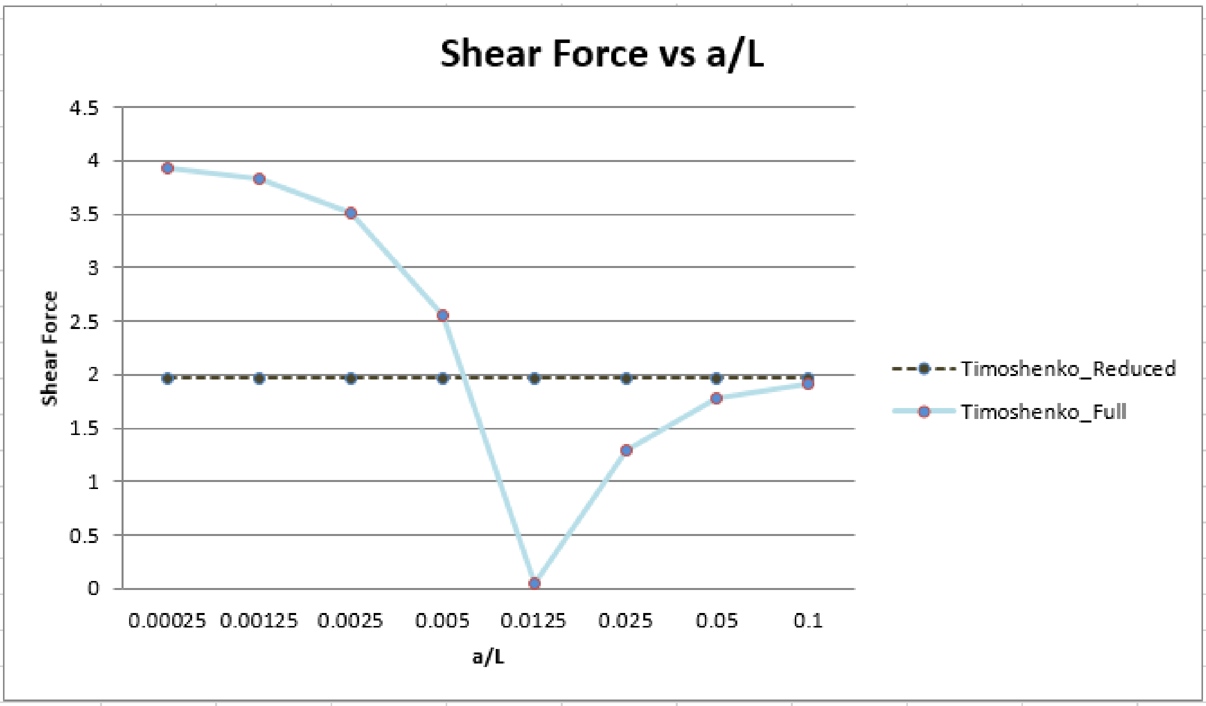

Figure 3: Plot of Shear force versus (a/l)

When the simulation of Shear force were analysed,for which the analytical result is '$2N$',and when the analytical result was compared with the numerical solution,we found that 'Timoshenko reduced integration' is very close to analytical solution.Euler-Bernoulli is not considered since as per their theory,no shear force is considered in this method. But while in other hand 'Timoshenko full integration' method behaves abruptly as we can observe from plot(Figure 3), i.e for lower (a/l) shear force is crossing analytical value. And as we increase the value of (a/l) or thickness,the shear force with this method also converges to analytical solution.As per theory we know that this method does not provide good accuracy,which can be seen in plots as shown above.

Finally we can conclude that Full integrated Timoshenko beam produces shear locking when solving a thin beam model. But using a reduced integration over beam takes care of the locking effect.And also we can say that Timoshenko full integration of beams gives correct results for high values of 'a' i.e, in other words gives better results for thicker beams.And Euler Bernoulli method gives the better results for variation of 'a'.

Note:The exact and numerical values calculated for Deflection(max),Moment and Shear force are shown below for reference:

Euler Bernoulli							
	a	a/L	W(max)			Moment	
			actual	obtained		actual	obtained
	0.001	0.00025	1.90E+09	1.90E+09		2	1.99
	0.005	0.00125	3.05E+06	3.05E+06		2	2.003
	0.01	0.0025	1.90E+05	1.90E+05		2	2.003
	0.02	0.005	1.19E+04	1.19E+04		2	2.003
	0.05	0.0125	3.04E+02	3.04E+02		2	2.003
	0.1	0.025	19.04	19.04		2	2.003
	0.2	0.05	1.19	1.19		2	2.003
	0.4	0.1	0.0744	0.0744		2	2.003

Figure 4: Actual and Numerical Values obtained for Euler-Bernoulli

Timoshenko reduced integration									
	a	a/L	W(max)			Moment		Q(Shear Force)	
			actual	obtained		actual	obtained	actual	obtained
	0.001	0.00025	1.90E+09	1.90E+09		2	1.99	2	1.9688
	0.005	0.00125	3.05E+06	3.05E+06		2	1.99	2	1.9688
	0.01	0.0025	1.90E+05	1.90E+05		2	1.99	2	1.9688
	0.02	0.005	1.19E+04	1.19E+04		2	1.99	2	1.9688
	0.05	0.0125	3.04E+02	3.04E+02		2	1.99	2	1.9688
	0.1	0.025	19.04	19.048		2	1.99	2	1.9688
	0.2	0.05	1.19	1.1905		2	1.99	2	1.9688
	0.4	0.1	0.0744	0.074405		2	1.99	2	1.9688

Figure 5: Actual and Numerical Values obtained for Timoshenko Reduced Integration

Timoshenko Full Integration									
	a	a/L	W(max)			Moment		Q(Shear Force)	
			actual	obtained		actual	Obtained	actual	Obtained
	0.001	0.00025	1.90E+09	1.46E+06		2	0.00153	2	3.933
	0.005	0.00125	3.05E+06	5.74E+04		2	0.0376	2	3.8262
	0.01	0.0025	1.90E+05	1.36E+04		2	0.1425	2	3.5162
	0.02	0.005	1.19E+04	2.80E+03		2	0.4697	2	2.5495
	0.05	0.0125	3.04E+02	2.00E+02		2	1.3144	2	0.0539
	0.1	0.025	19.04	1.69E+01		2	1.7687	2	1.2883
	0.2	0.05	1.19	1.16E+00		2	1.936	2	1.7825
	0.4	0.1	0.0744	7.55E-02		2	1.982	2	1.921

Figure 6: Actual and Numerical Values obtained for Timoshenko Full Integration