COMPUTATIONAL SUCTURAL MECHANICS AND DYNAMICS

Master of Science in Computational Mechanics/Numerical Methods

Spring Semester 2019

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Assignment 6: Bending of beams

1. **Program in MatLab the Timoshenko 2 Nodes Beam element with reduced integration for the shear stiffness matrix:**

   In order to implement the reduced integration, the elemental shear stiffness matrix has been changed as follows:

```
K_s = [  1    ,   len/2  ,   -1    ,   len/2  ;
       len/2  , len^2/4  , -len/2  , len^2/4  ;
        -1    ,  -len/2  ,    1    ,  -len/2  ;
       len/2  , len^2/4  , -len/2  ,  len^2/4 ];
```

2. **Solve the following problem with a 64 element mesh with the:**
   - **2 nodes Euler Bernoulli element**
   - **2 nodes Timoshenko Fully Integrated element**
   - **2 nodes Timoshenko Reduced Integration element**

   Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.

   Eight different relations of a/L have been computed with the three types of element. To compare the results, the Timoshenko Reduced Integration element has been chosen as reference:

   The maximum deflection, bending moment and shear force of the beam for different a/L relation are plotted in the following figures:

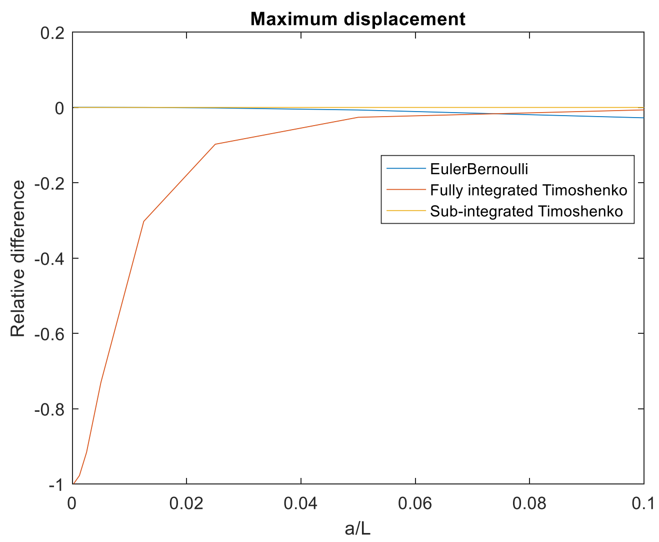

Figure 1: Maximum displacement for different elements

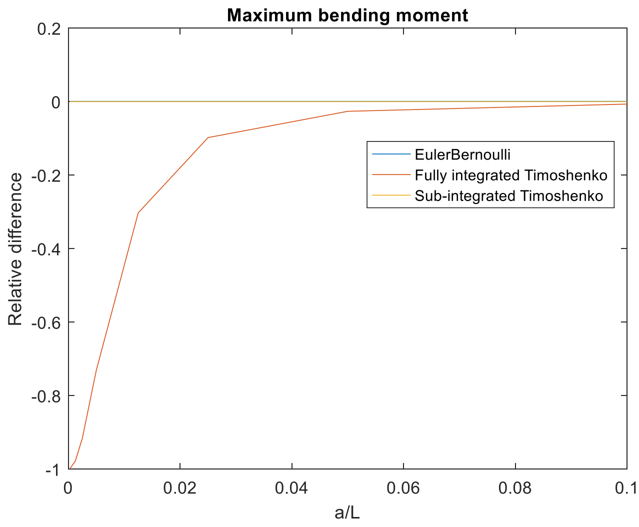

Figure 2: Maximum bending moment for different elements

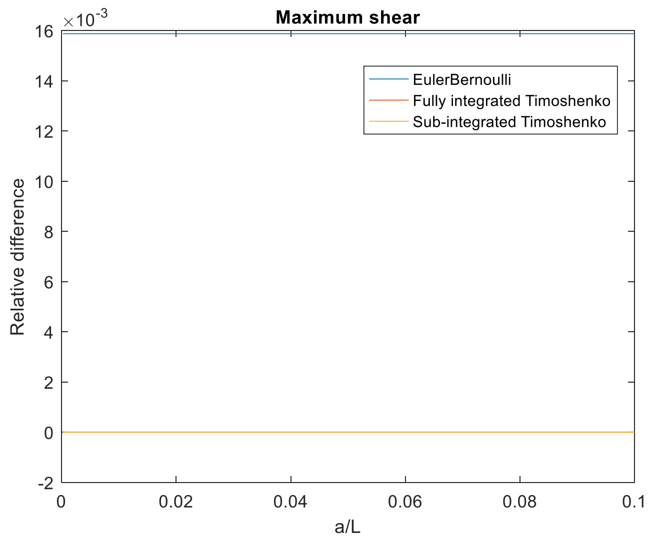

Figure 3: Maximum shear for different elements

From this plots, it is seen that for low values of a/L, the Euler Bernoulli and the sub-integrated Timoshenko beam yield to the same results while the fully integrated Timoshenko is completely stiff due to shear locking. Due to this phenomenon, the bending moment is highly underestimated. As the relation a/L increases, the shear effects become more significant. For this reason, the Timoshenko beam is more accurate. The interesting point is that, in this case, the sub-integrated element converges to the same result than the fully integrated element while the deflection of the Euler Bernoulli beam is higher than what it should due to the not consideration of the shear effects. However, even in this case, the Euler Bernoulli beam leads to correct values of the bending moment.

About the shearing force, all of the three beams provide accurate results.