Assignment 7
Computational Structural Mechanics & Dynamics
BEAMS

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Q.1a.

We modify the code in Matlab to obtain Timoshenko 2 Nodes Beam element with reduced integration. We modify in the code the following stiffness matrix:

Bending stiffness matrix is coded as :

$$K_b^{(e)} = \left(\frac{EI}{l}\right)^{(e)} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

The shear stiffness matrix is coded as:

$$K_s^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ & \frac{{l^{(e)}}^2}{4} & -\frac{l^{(e)}}{2} & \frac{{l^{(e)}}^2}{4} \\ & & 1 & -\frac{l^{(e)}}{2} \\ Symm. & & & \frac{{l^{(e)}}^2}{4} \end{bmatrix}$$

One gauss point is taken as 0 for stress evaluation.

Images of the modified code are given below:

```
const = D_matb/len;

K_b  = [ 0 ,  0 ,  0 ,  0 ;
         0 ,  1 ,  0 , -1 ;
         0 ,  0 ,  0 ,  0 ;
         0 , -1 ,  0 ,  1 ];

K_b  = K_b * const;

const =   D_mats/len;

K_s = [   1    ,    len/2 ,    -1    ,    len/2 ;
           len/2 , len^2/4 , -len/2 , len^2/4 ;
            -1    ,  -len/2 ,     1    ,  -len/2 ;
           len/2 , len^2/4 , -len/2 ,  len^2/4 ];

K_s = K_s * const;
```

```
% One gauss point for stress evaluation
gaus0 =  0.0;    % One Gauss point for stresses evaluation

bmat_b=[ 0, -1/len, 0, 1/len];

bmat_s1=[-1/len,-(1-gaus0)/2, 1/len,-(1+gaus0)/2];

Str1_g0 = D_matb*(bmat_b *transpose(u_elem));
Str2_g0 = D_mats*(bmat_s1*transpose(u_elem));

Strnod(lnods(1),1) = Strnod(lnods(1),1)+Str1_g0;
Strnod(lnods(2),1) = Strnod(lnods(2),1)+Str1_g0;
Strnod(lnods(1),2) = Strnod(lnods(1),2)+Str2_g0;
Strnod(lnods(2),2) = Strnod(lnods(2),2)+Str2_g0;
Strnod(lnods(1),3) = Strnod(lnods(1),3)+1;
Strnod(lnods(2),3) = Strnod(lnods(2),3)+1;
```

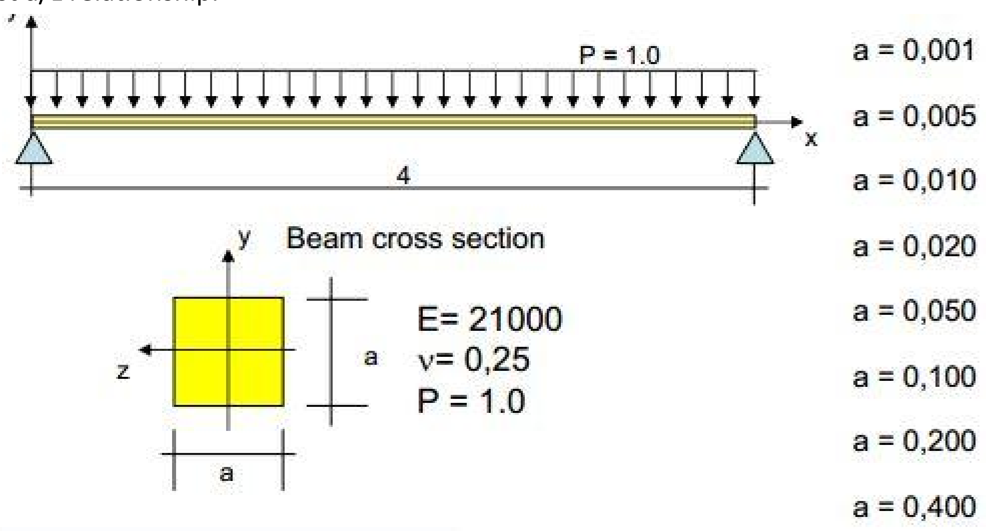
The discussions of the modified code is done in the next section.

Q.1.b.

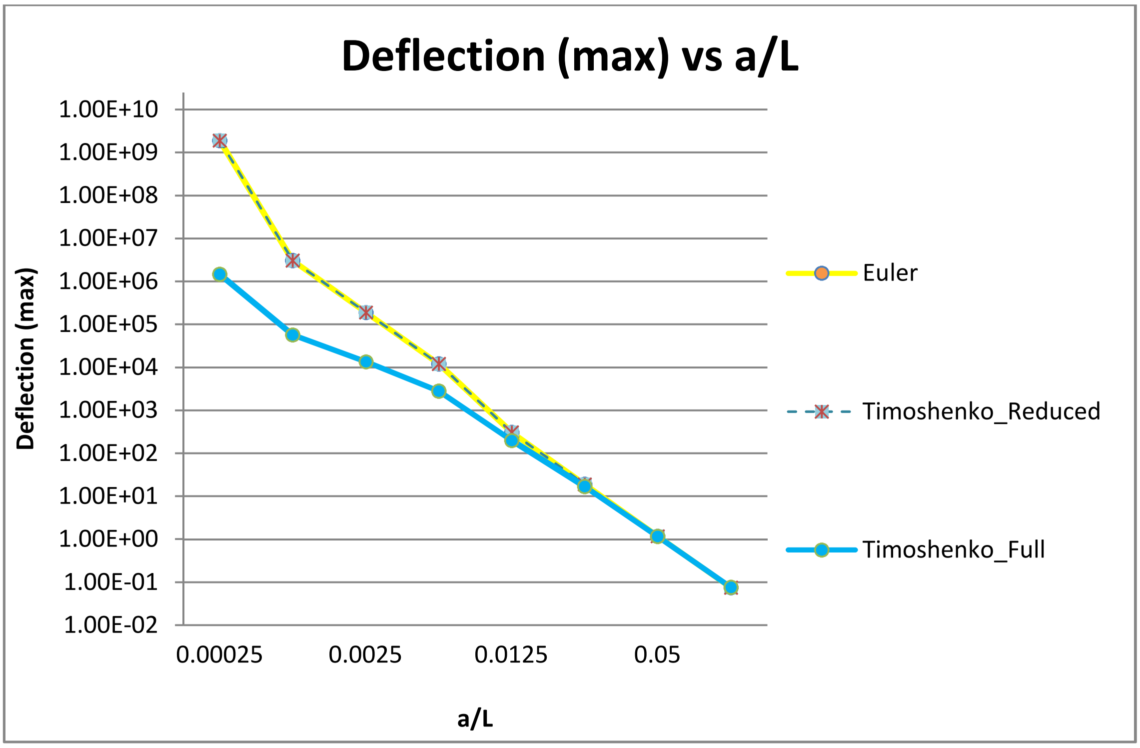
The following problem with a 64 element mesh with

- 2 nodes Euler element
- 2 nodes Timoshenko Integrate element
- 2 nodes Timoshenko Reduced Integration element

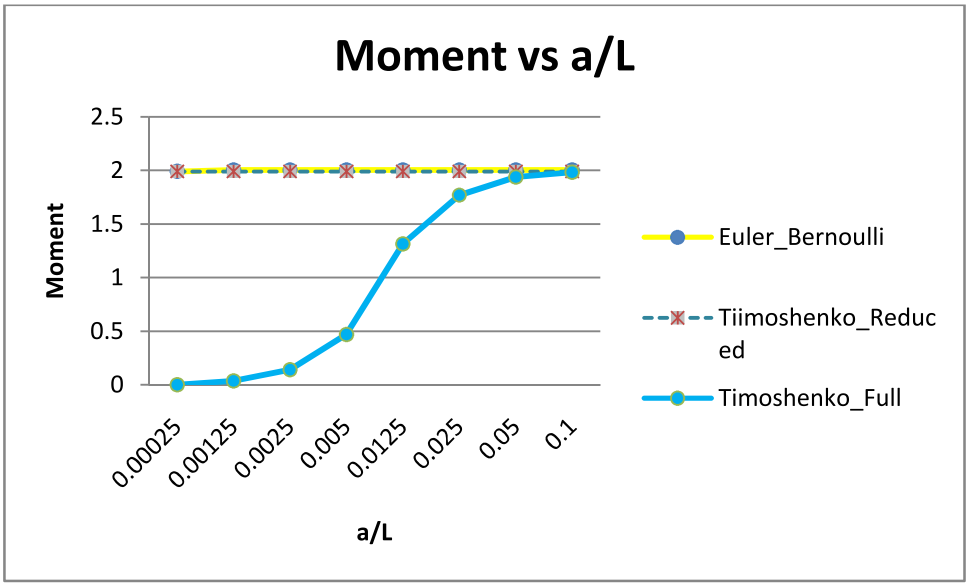
is solved and the displacements, moments and shear for the 3 elements are compared against a/L relationship.



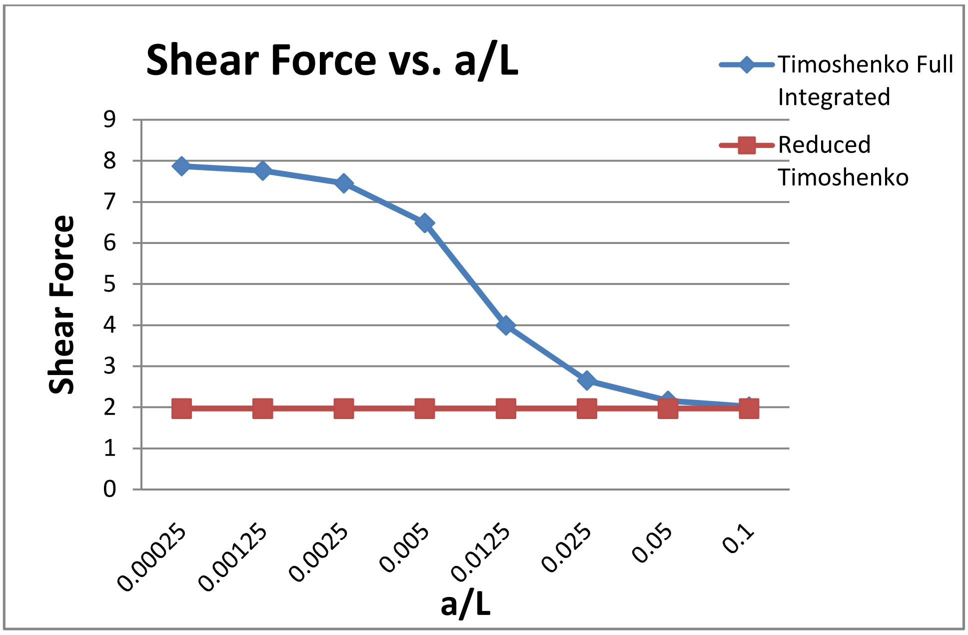
The following graphs are plotted and the table of values are provided for better understanding:



From the above graph it is evident that displacements in Euler element and reduced integration Timoshenko element are same. As the beam becomes more thicker, the displacement in full integration Timoshenko element start to converge to give accurate result. The error of actual value of displacements is very low for Euler element and also for reduced Timoshenko element. But for thick beam, the error of displacement for reduced Timoshenko is little higher. The errors are calculated in the table provided at the end of the report.



The analytical value of moment is calculated to be 2 Nm. For Euler and reduced Timoshenko beam, the moment values are very close to the analytical value for overall thickness of the beam. For very thin beam, the moment calculated in full integrated Timoshenko beam is very far from the analytical value and it slowly converges to it as the beam becomes thicker as shown in the figure above.



The shear stress is constant for reduced Timoshenko beam whereas the shear stress is very high for full integrated Timoshenko beam at very low thickness and it gradually converges as the beam becomes thicker. In Euler beams, shear stress is not considered unlike in Timoshenko beams. This is due to the fact that in Euler beams bending is assumed to behave in such a way that cross section normal to neutral axis remains normal after bending, whereas in Timoshenko beams, it does not remain after bending.

Table of computed values :

Euler							
a	disp(obtained)	disp(actual)	moment(obt)	moment(actual)	a/L	error(disp)
0.001	1.9048E+09	1.90E+09	2.0003	2	0.00025	0.00E+00
0.005	3.0476E+06	3.05E+06	2.0003	2	0.00125	1.97E-02
0.01	1.9048E+05	1.90E+05	2.0003	2	0.0025	4.20E-02
0.02	1.1905E+04	1.19E+04	2.0003	2	0.005	4.20E-02
0.05	3.0476E+02	3.04E+02	2.0003	2	0.0125	2.50E-01
0.1	1.9048E+01	19.04	2.0003	2	0.025	4.20E-02
0.2	1.1905E+00	1.19	2.0003	2	0.05	4.20E-02
0.4	7.4405E-02	0.0744	2.0003	2	0.1	6.72E-03

Table 1.

Reduced Timoshenko								
a	disp(obtained)	disp(actual)	M (obt)	M (actual)	a/L	Q actual	Q (obt)	Error(disp)
0.001	1.9040E+09	1.90E+09	1.999	2	0.00025	2	1.9688	4.20E-02
0.005	3.0464E+06	3.05E+06	1.999	2	0.00125	2	1.9688	1.97E-02
0.01	1.9041E+05	1.90E+05	1.999	2	0.0025	2	1.9688	5.25E-03
0.02	1.1901E+04	1.19E+04	1.999	2	0.005	2	1.9688	8.40E-03
0.05	3.0476E+02	3.04E+02	1.999	2	0.0125	2	1.9688	2.50E-01
0.1	1.9069E+01	19.04	1.999	2	0.025	2	1.9688	1.52E-01
0.2	1.1971E+00	1.19	1.999	2	0.05	2	1.9688	5.97E-01
0.4	7.6161E-02	0.0744	1.999	2	0.1	2	1.9688	2.37E+00
Q=Shear force, Moment=M								

Table 2.

Timoshenko Full Integrated				
a	disp(obtained)	moment(obt)	a/L	Shear force obtained
0.001	1461500.0000	7.8705	0.00025	7.8705
0.005	57401.0000	7.7637	0.00125	7.7637
0.01	13583.0000	7.4538	0.0025	7.4538
0.02	2797.3000	6.487	0.005	6.487
0.05	200.4300	3.9916	0.0125	3.9914
0.1	16.8750	2.6497	0.025	2.6492
0.2	1.1596	2.1558	0.05	2.1549
0.4	0.0756	2.0174	0.1	2.0164

Table 3.