

Computational Structural Mechanics and Dynamics - Practice 5

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1 Plane Frame

1.1 Introduction

In this first exercise, a bar structure has to be analyzed via modal analysis. The structure is clamped on the base supports and no displacement is allowed in the z direction.

A first approach to obtain the natural frequencies is required, followed by the study of the effect of different weighted dynamic load frequencies in the upper left corner of the structure.



Figure 1: Problem statement of the first exercise.

The direct time integration will be carried with these different excitation inputs in(1) :

$$q = A \sin(\omega_P t) \quad (1)$$
$$A = 50kN \quad \omega_P = [0.75\omega_1; 1.0\omega_1; 1.25\omega_1]$$

1.2 Preprocess

The structure has been modeled in Tdyn with different bar section and linear elements. Table 1 .

	Nodes	Elements
Mesh	59	60

Table 1: Mesh used in the modal and direct time analysis.

Once the mesh was created we proceeded to the modal analysis to find the natural frequencies of the structure and then to the dynamic analysis with direct time integration.

1.3 Modal analysis

For the modal analysis we run Tdyn without any external load and with the predefined option of Modal Analysis. We computed the first 10 natural frequencies and its respective modes. The natural frequencies are summarized in Table 2 and the modes in Figure 2 and 3

Mode	Freq [Hz]
1	4.214
2	12.51
3	19.530
4	43.14
5	48.01
6	52.34
7	54.79
8	54.79
9	54.79
10	73.89

Table 2: Natural frequencies of the first 10 modes of the structure.

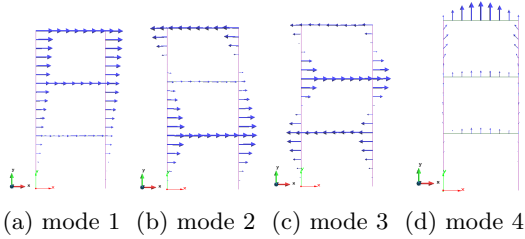


Figure 2: Shape of the first 4 modes of the structure.

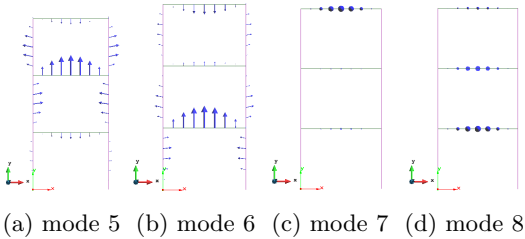


Figure 3: Shape of the second 4 modes of the structure.

1.4 Dynamic analysis

For the dynamic analysis the structure has to be excited with a periodic sinusoidal load given by Equation (1) applied at the upper shell as seen in Figure 1. The three frequencies that define the load can now be computed with the first natural frequency of the structure $\omega_1 = 4.214\text{Hz}$. To be able to capture the dynamic response of the structure, the time steps used for the direct time integration have to be smaller than the oscillation period of the natural frequency of resonance. To ensure this the time step has been defined to be one tenth of the excitation period (the inverse of the frequency), in our case we have chosen these time to be 0.02s. The Sinusoidal load definition in Tdyn by default is entered as $2\pi\omega_P$ so in order to well define load case that we are asked to model, the natural frequencies computed in the modal analysis have to be divided by 2π to be introduced in the shell pressure load. All these input values are summarized in Table 3.

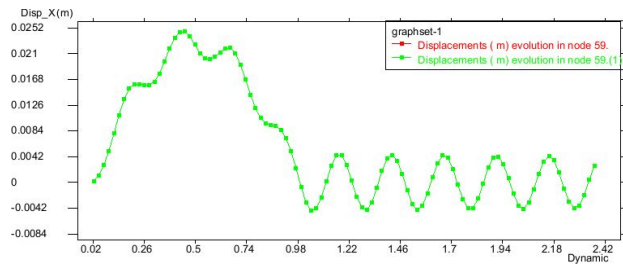
	ω_i [Hz]	Period [s]	$\omega_i/2\pi$ [Hz]
Case 1	3.1605	0.316	0.503
Case 2	4.214	0.237	0.670
Case 3	5.2675	0.1898	0.838

Table 3: Frequencies of the periodic load.

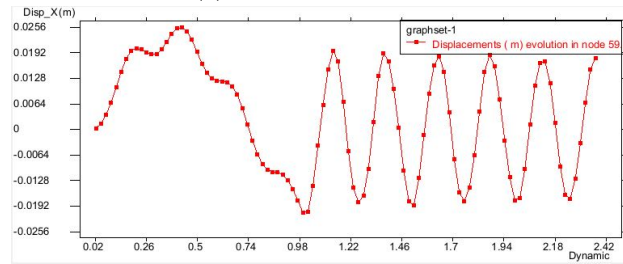
It can be easily appreciated how the dynamic response varies with the frequency, being the higher results when we are in the First mode frequency. This results are logical, as our structure is resonating at that point.

It has to be said, though, that for all three responses there is an initial phase adaptation to the sinusoidal wave response. With these results

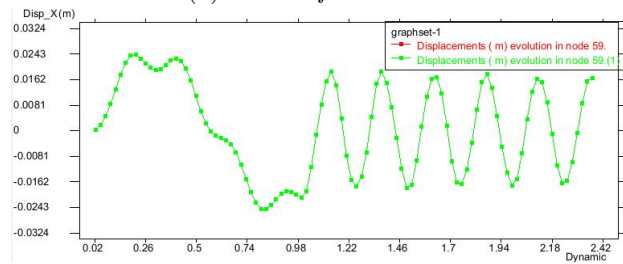
we confirm the theory of the subject when it comes to modal analysis and direct time integration.



(a) Case 1. $f = 3.16$ Hz



(b) Case 2. $f = 4.214$ Hz



(c) Case 3. $f = 5.267$ Hz

Figure 4: X-direction displacement versus time computed by direct time integration.

2 Spatial shell

2.1 Introduction

In this second exercise we want to study the natural frequencies and modes of an angular spatial shell under a periodic load at the surface (Figure 5). The structure is clamped at the lower edge and left free in all the other boundaries. We will carry a modal analysis and also with a direct time integration method. For the dynamic analysis the structure is perturbed in three cases with a periodic load as function of the first natural frequency given by (2) .

$$q = A \sin(\omega_P t)$$

$$A = 50 \frac{kN}{m^2} \quad \omega_P = [0.75\omega_1; 1.0\omega_1; 1.25\omega_1] \quad (2)$$

The structure is made out of concrete with the material properties of Table 4.

Material	E	ν	γ	t
HA-35	$3.0e10 \frac{N}{m^2}$	0.25	$25 \frac{N}{m^3}$	0.3m

Table 4: Properties of the shell structure.

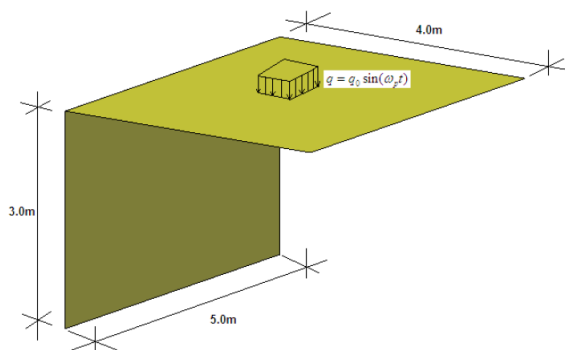


Figure 5: Problem statement of the second exercise.

2.2 Preprocess

The structure has been modeled in Tdyn with two shells and meshed with linear triangle shell elements (Table 5).

	Nodes	Elements
Mesh	341	614

Table 5: Mesh used in the modal and direct time analysis.

Once the mesh was created we proceeded to the modal analysis to find the natural frequencies of the structure and then to the dynamic analysis with direct time integration.

2.3 Modal analysis

For the modal analysis we run Tdyn without any external load and with the predefined option of Modal Analysis. We computed the first 10 natural frequencies and its respective modes. The natural frequencies are summarized in Table 6 and the modes in Figure 6.

Mode	Frequency [Hz]
1	4.551
2	8.747
3	12.73
4	19.94
5	51.33
6	57.75
7	72.96
8	83.32
9	105.7
10	116.9

Table 6: Natural frequencies of the first 10 modes of the shell structure.

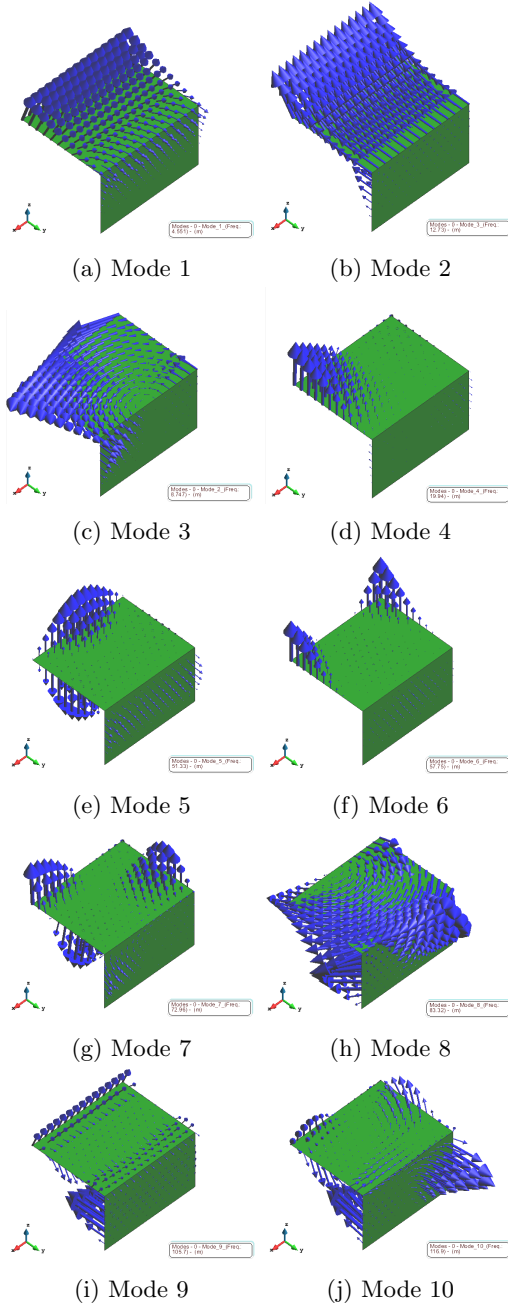


Figure 6: Shape of the first 10 modes of the structure.

2.4 Dynamic analysis

For the dynamic analysis as we said the structure has to be excited with a periodic sinusoidal load given by Equation (2) applied at the upper shell as seen in Figure 5. The three frequencies that define the load can now be computed with the first natural frequency of the structure $\omega_1 = 4.551\text{Hz}$.

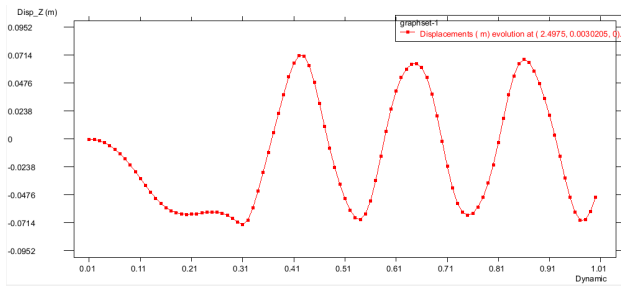
To be able to capture the dynamic response of the structure, the time steps used for the direct time integration have to be smaller than the oscillation period of the natural frequency of resonance. To ensure this the time step has been defined to be one tenth of the excitation period (the inverse of the frequency), in our case we have chosen these time to be 0.01s. The Sinusoidal load definition in Tdyn by default is entered as $2\pi\omega_P$ so in order to well define load case that we are asked to model, the natural frequencies computed in the modal analysis have to be divided by 2π to be introduced in the shell pressure load. All these input values are summarized in Table 7.

	ω_i [Hz]	Period [s]	$\omega_i/2\pi$ [Hz]
Case 1	3.4133	0.292976	0.5432
Case 2	4.5510	0.219732	0.7243
Case 3	5.6888	0.175786	0.9054

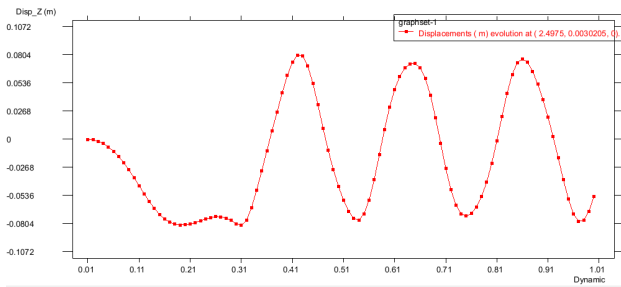
Table 7: Frequencies of the periodic load.

To see the dynamic response of the structure, we have created a time versus z-displacements graphs tracking the mid point of the external top end of the angular profile. As we can see in Figure 7 in all three cases the structure after an initial phase of adaption to the dynamic load, starts oscillating with a period of about 0.22s that corresponds to the first natural frequency

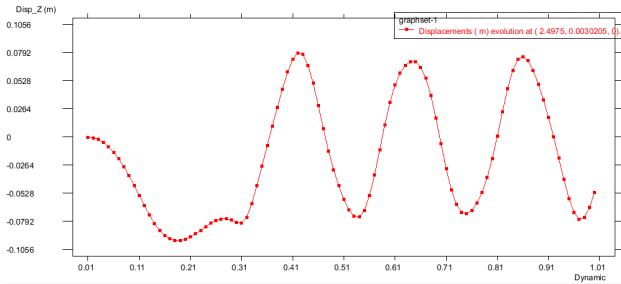
of 4.551 Hz. In the specific case of Figure 7b that corresponds to a periodic load of exactly the first natural frequency we observe that the amplitude of the oscillation is higher than the one of the two other cases, even than the one of the third case that has a higher maximum prescribed displacement. This corresponds to the resonance of the structure in the first natural frequency and mode.



(a) Case 1. $f = 3.4133$ Hz



(b) Case 2. $f = 4.5510$ Hz



(c) Case 3. $f = 5.6888$ Hz

Figure 7: Vertical displacement versus time computed by direct time integration.