Universitat Politècnica de Catalunya

Master of Science in Computational Mechanics

Computational Structural Mechanics and Dynamics

Practice 2
3D Solids

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March 29, 2019

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1 Circular tank

The figure shows a circular tank made of reinforced concrete. It is used for the storage of water in a water purification plant. Analyze the structural behavior of the tank. Use quadrilateral elements with four nodes.

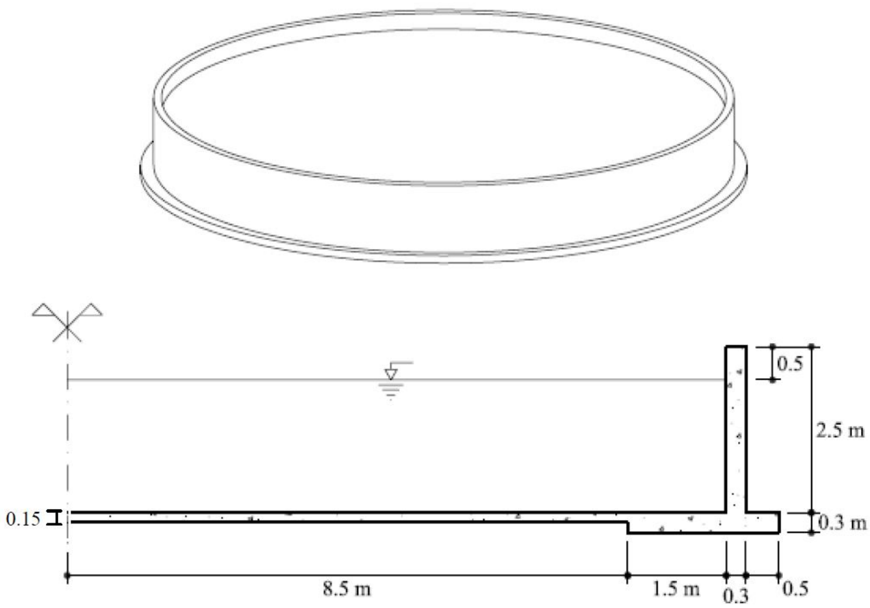


Figure 1.1: Exercise 1

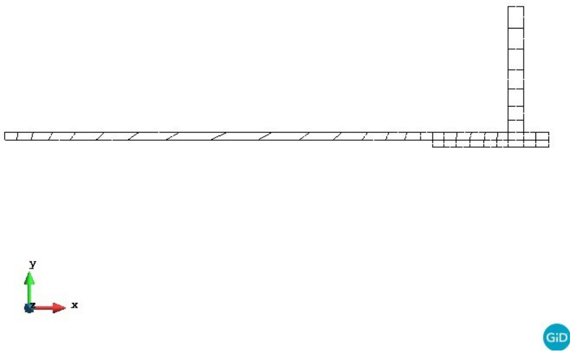
1.1 Problem data:

$$\text{Concrete}\begin{cases} E = 3.0e10\dfrac{N}{m^2} \\ \nu = 0.2 \end{cases}$$

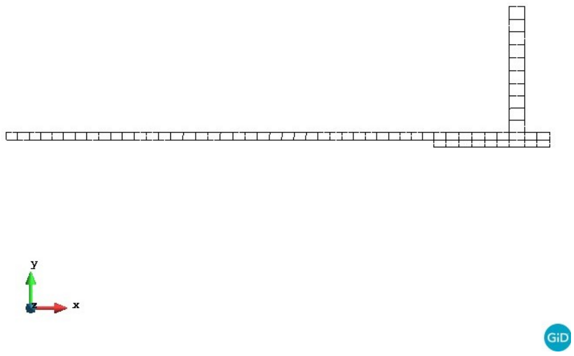
$$\text{Floor}\left\{\text{Ballast coefficient} = 50\dfrac{N}{cm^3}\right.$$

1.2 Discretization

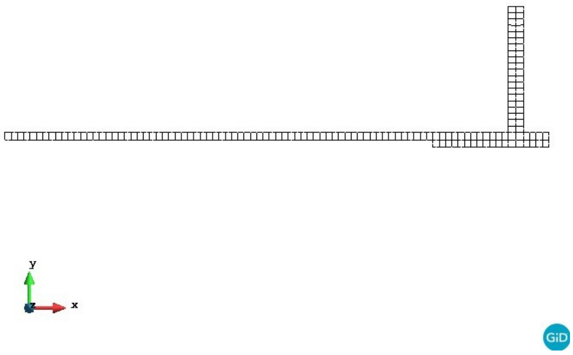
In order to evaluate the structural behavior and be able to make convergence graphics 5 different types of discretization using quadrilateral elements were proposed.



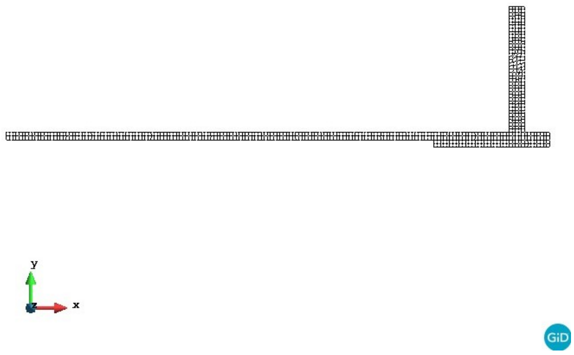
(a) Mesh 1



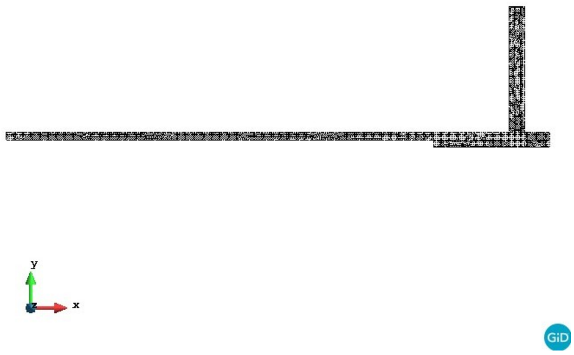
(b) Mesh 2



(c) Mesh 3



(d) Mesh 4



(e) Mesh 5

Figure 1.2: Discretization

1.3 Postprocessing

The comparison of the different types of meshes used will be made from the evaluation of the displacements in the x-direction and of the displacements in the y-direction at a point belonging to the structure which is going to be the same through all the meshes.

Here, by way of example, the graphs of displacement in x-direction and displacement in y-direction of the mesh 2 are shown:

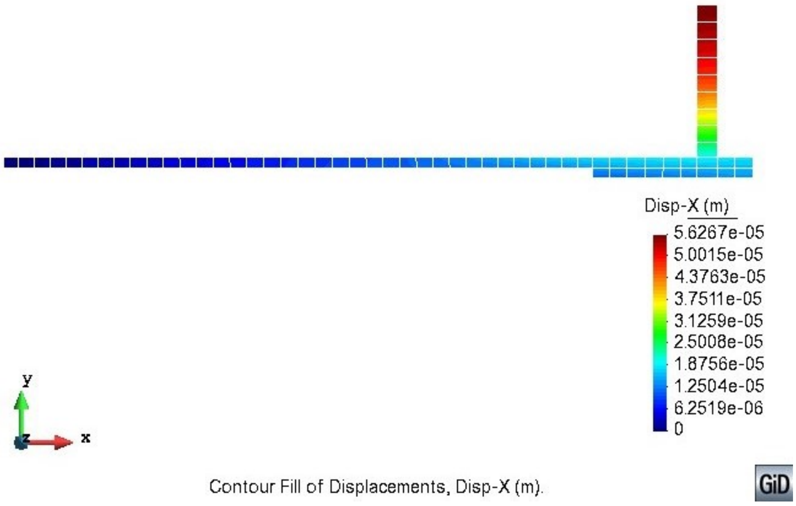


Figure 1.3: Displacement in x-direction

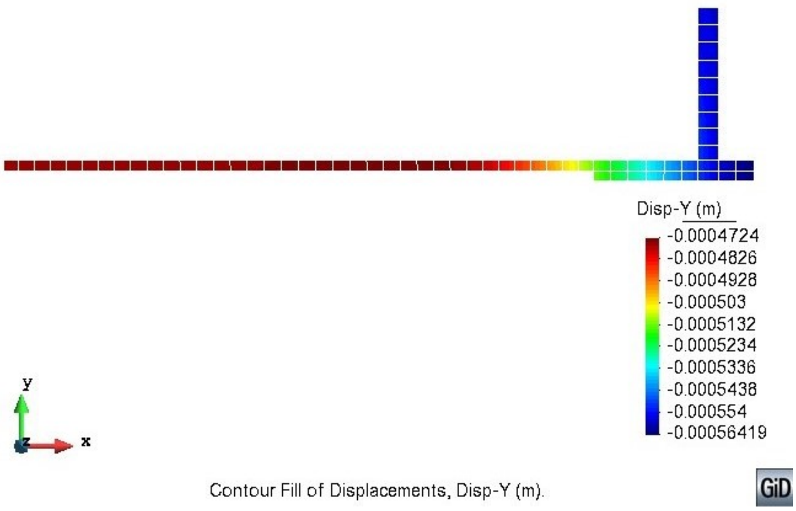


Figure 1.4: Displacement in y-direction

As it was said before, the convergence graph will be shown for the same point at the displacement in the x and y direction, the following table shows the data collected for the different meshes:

Nº	Maximum element size	Elements	Nodes	Displacement in x-direction	Displacement in y-direction
Mesh 1	0.5	29	56	4,46E-05	-3,07E-04
Mesh 2	0.25	68	129	5,02E-05	-2,91E-04
Mesh 3	0.125	274	394	5,18E-05	-2,83E-04
Mesh 4	0.0625	904	1143	5,20E-05	-2,81E-04
Mesh 5	0.03125	3150	3624	5,22E-05	-2,80E-04

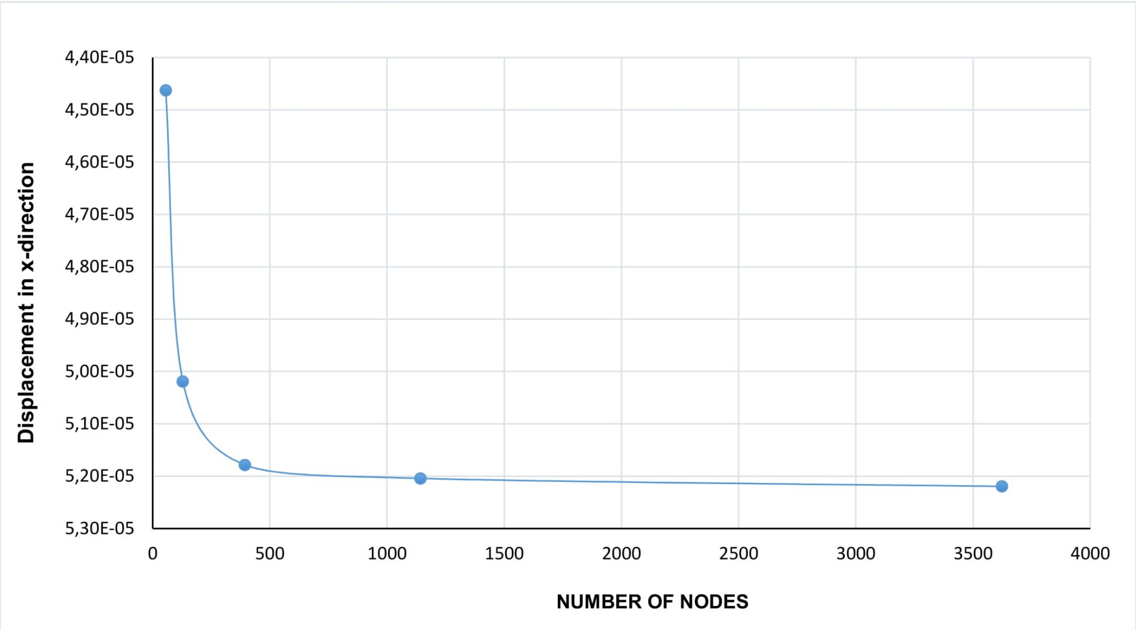


Figure 1.5: Convergence graph (Displacement in x-direction)

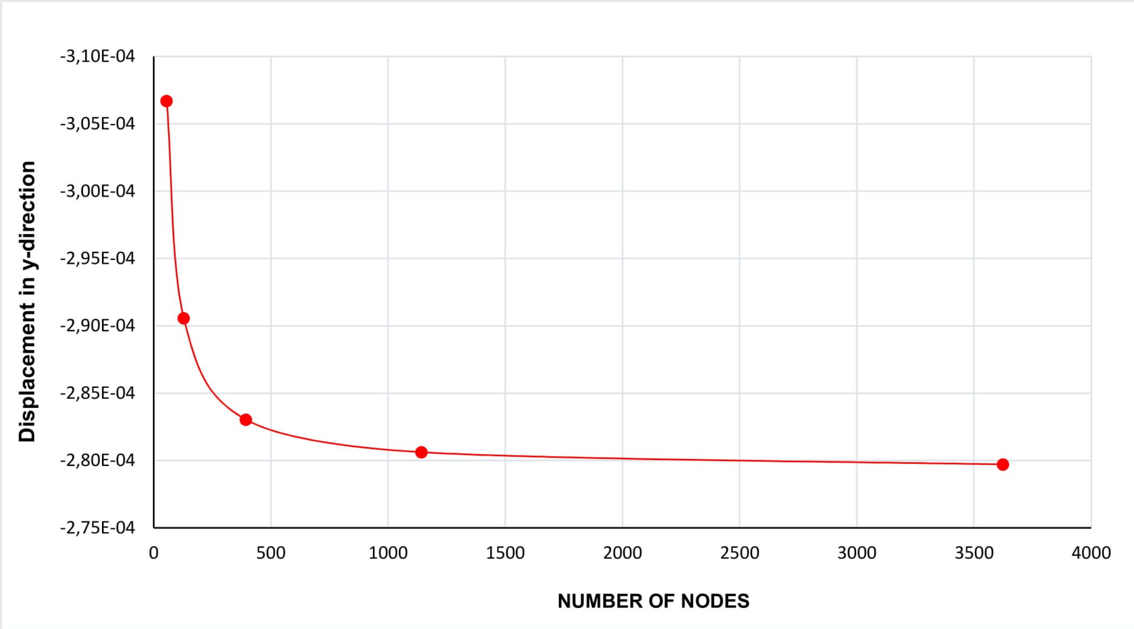


Figure 1.6: Convergence graph (Displacement in y-direction)

2 Flexion of beam

The cantilever depicted on Figure 2.1 is under the action of a moment at it's free edge. To evaluate the flexion of the beam via finite elements, a 3D model was conceived with the fixed surface being prescribed zero displacement and the moment applied through a conjugate force pair of $P = 10kN$ as also shown on Figure 2.1.

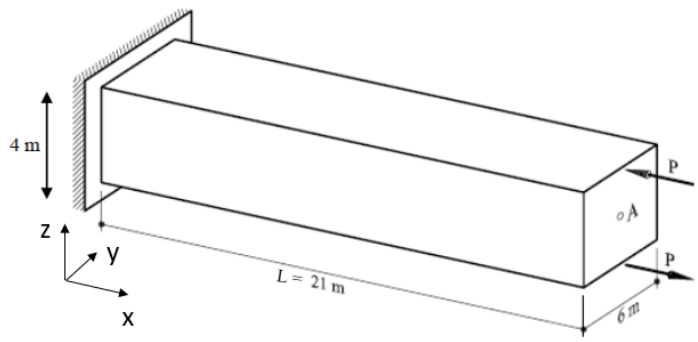


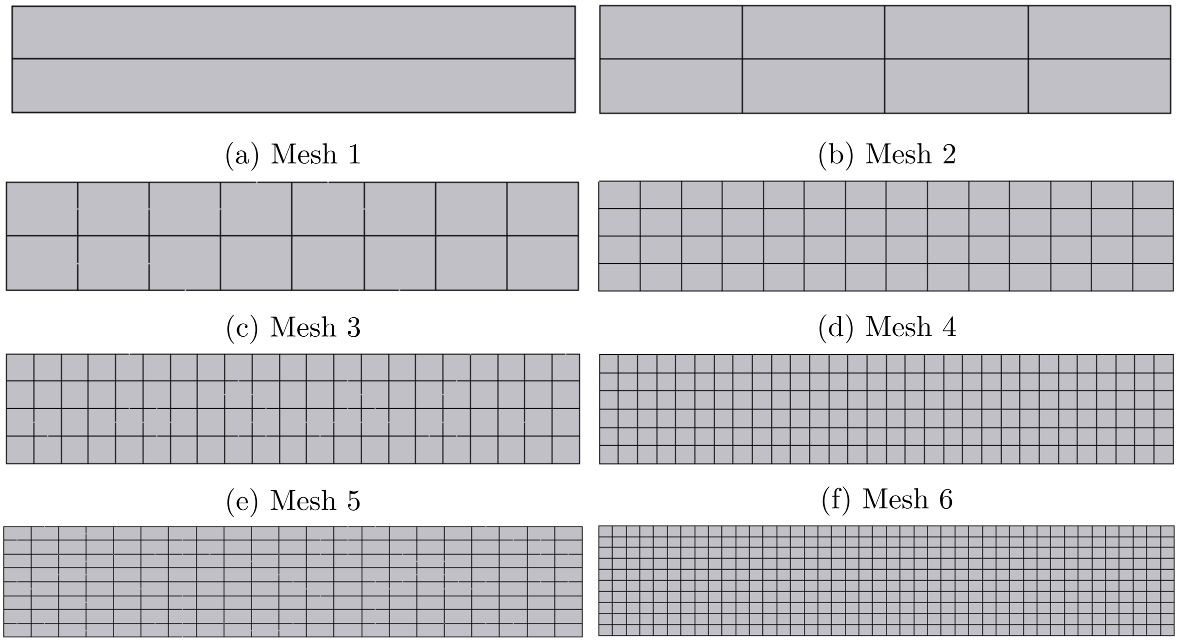
Figure 2.1: Problem geometry and loading

According to the Euler-Bernoulli beam theory, the Z-displacement of the elastic line of a beam under the described conditions follows Equation 2.1:

$$w(x) = \frac{Mx^2}{EI} \tag{2.1}$$

where M is the Moment, L is the beam's length, E is the elastic modulus and I is the moment of inertia of the bar.

To compare the beam theory with the FEM solution, eight meshes were made, using hexahedra elements with both linear and quadratic interpolation. The different grids can be seen on Figure 2.2 and are listed on Table 2.1 alongside with the number of elements and nodes. For the comparison of the results, the results of Z-displacement on the free edge of the vertical line were chosen, given that the Beam theory is only valid for the elastic line and the free edge will be subject to the greatest displacement.



(a) Mesh 1 (b) Mesh 2
(c) Mesh 3 (d) Mesh 4
(e) Mesh 5 (f) Mesh 6
(g) Mesh 7 (h) Mesh 8

Figure 2.2: Discretization on the X-Z plane

The results for each mesh listed on Table 2.1 and plotted on Figure 2.3. As observed, the linear and the quadratic models were able to eventually match the beam theory. However, the model with linear interpolation required over 500 elements (over 700 nodes) to reach a result with error of under 3% against the beam theory. On the other hand, with as little as 4 elements (51 nodes), the quadratic model already provides errors of 2%.

Table 2.1: Mesh properties for the convergence test

		Linear		Quadratic	
Mesh	no. of elem.	no. of nodes	w at x=L [m]	no. of nodes	w at x=L [m]
1	4	18	$1.05{\cdot}10^{-7}$	51	$1.29{\cdot}10^{-6}$
2	16	45	$7.51{\cdot}10^{-7}$	141	$1.31{\cdot}10^{-6}$
3	32	81	$1.09{\cdot}10^{-6}$	261	$1.31{\cdot}10^{-6}$
4	224	375	$1.23{\cdot}10^{-6}$	1325	$1.31{\cdot}10^{-6}$
5	504	770	$1.27{\cdot}10^{-6}$	2871	$1.32{\cdot}10^{-6}$
6	1440	1953	$1.29{\cdot}10^{-6}$	7253	$1.32{\cdot}10^{-6}$
7	2016	2574	$1.28{\cdot}10^{-6}$	9695	$1.32{\cdot}10^{-6}$
8	7560	8987	$1.31{\cdot}10^{-6}$	-	-

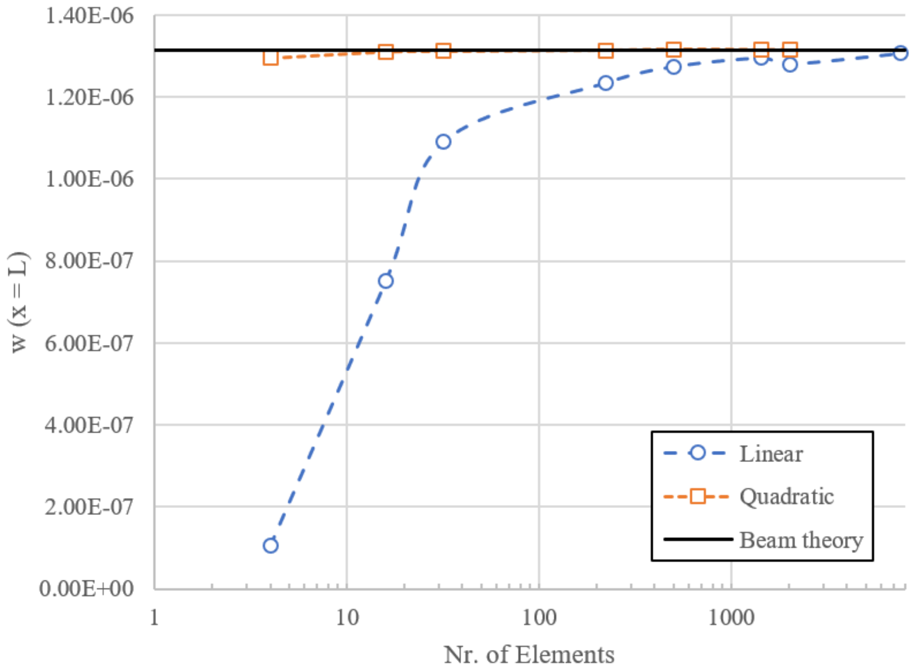


Figure 2.3: Convergence of the Z-displacement in respect to the number of elements

The effect of the moment on the cantilever is shown on Figure 2.4. The moment performs a deflection of the bar by compressing it's upper side while pulling the lower side at the same time.

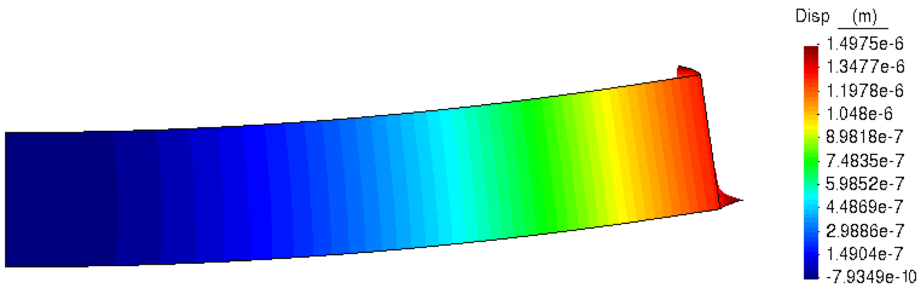


Figure 2.4: Deformation (with a 10^6 factor) of the bar and contour of the Z-Displacement

3 Foundation of a corner column

The figure shows a corner column with its foundation. This type of foundation is characterized by the fact that the support reactions are eccentric with respect to the load of the column. This results in a flexion of the column and lifting of the base slab.

Analyze the state of stress in the column and the slab under the assumption that the slab is supported elastically by the ground. Determine whether or not the slab suffers lifting. Use hexahedrons with eight nodes.

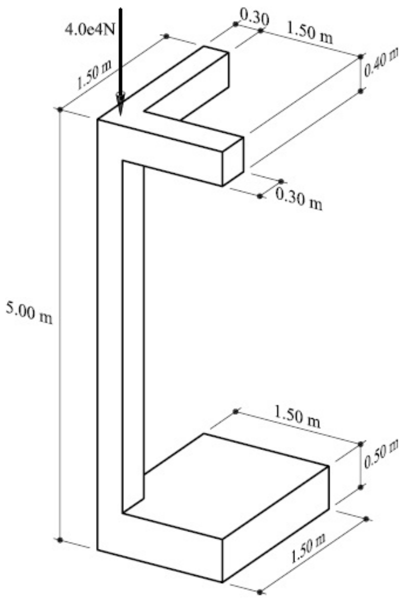
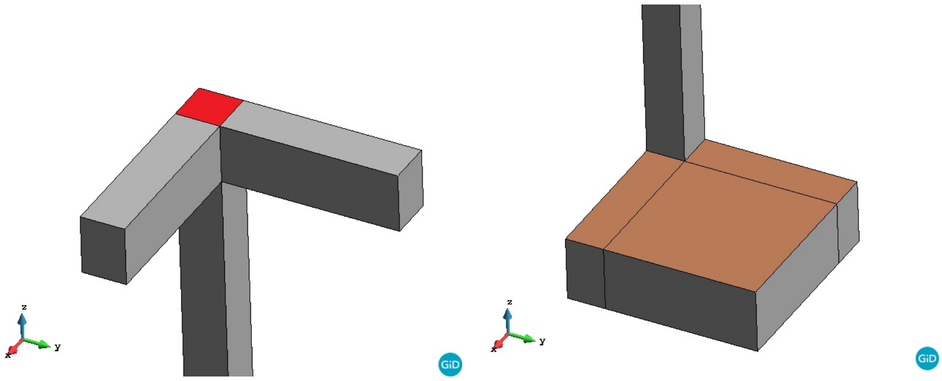


Figure 3.1: Problem geometry

Material properties: Concrete $\begin{cases} E = 3.0e10\dfrac{N}{m^2} \\ \nu = 0.2 \end{cases}$ Ground $\left\{ \text{Ballast coefficient} = 50\dfrac{N}{cm^3} \right.$

Boundary conditions: A uniformly distributed load over the red region (figure 3.2) of magnitude 40 KN in total is applied. To support the structure soil is poured over the shaded region (figure 3.2) onn the base to height of 1.5 m which in turn applies uniformly distributed load of $27KN/m^2$ (Density of soil $18KN/m^2$). Base is supported with ground from bottom side with elastic constraints (Ballast coefficient = 50 N/cm^3).



(a) Forces on top portion (b) Forces on bottom portion

Figure 3.2: Boundary conditions

The discretization process is carried out with the geometry by considering hexahedron elements as seen in figure 3.3. Resultant mesh consist of 5750 hexahedron elements.

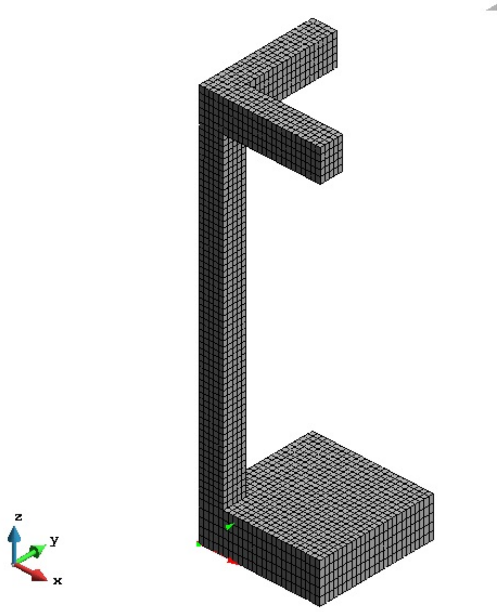


Figure 3.3: Discretization

After solving the problem following results were observed in the post processing part. The total Displacements observed can be seen in figure 3.4, maximum displacement observed was 5.9 mm which is significantly low as compared to size of the structure. In figure 3.5 it can be seen the displacement in Z direction is concentrated towards free corner opposite to the support.

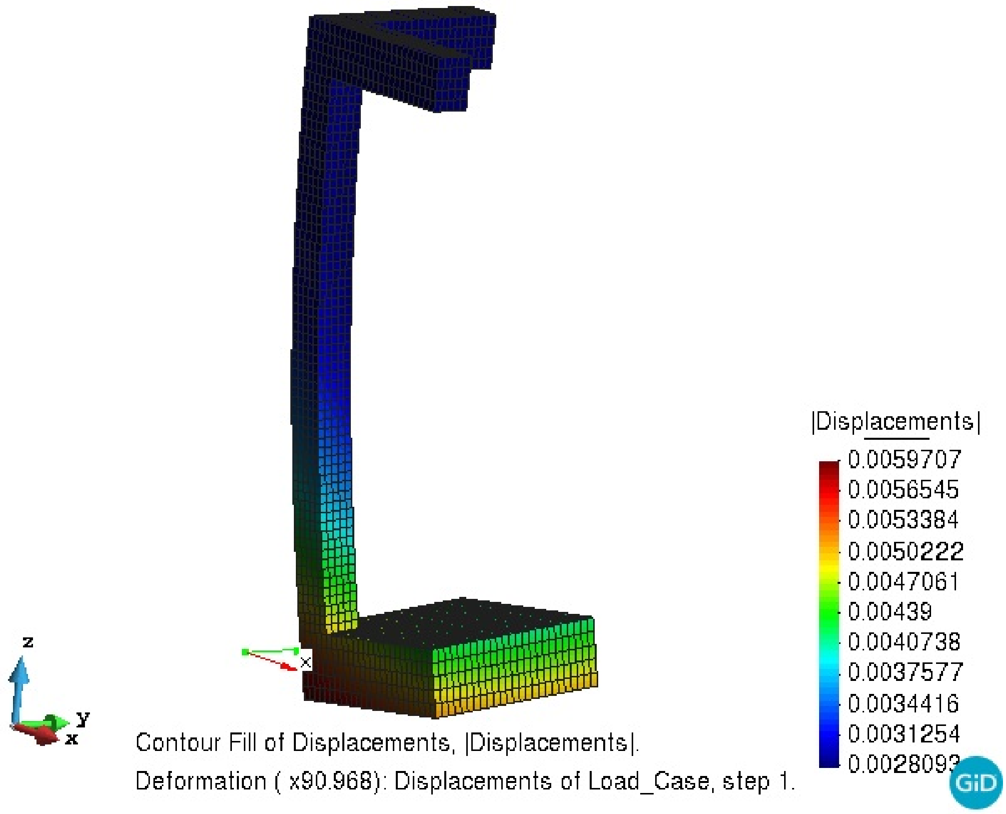


Figure 3.4: Total displacement

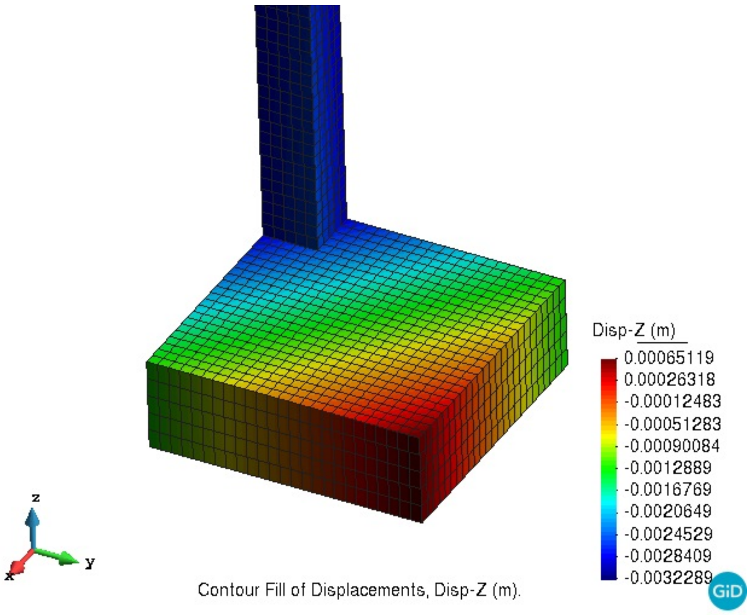


Figure 3.5: Displacement in Z direction

Following plots for the normal stress in Z (figure 3.6) direction and Von misses stress (figure 3.7) the maximum stress observed were under elastic limit thus can be considered safe.

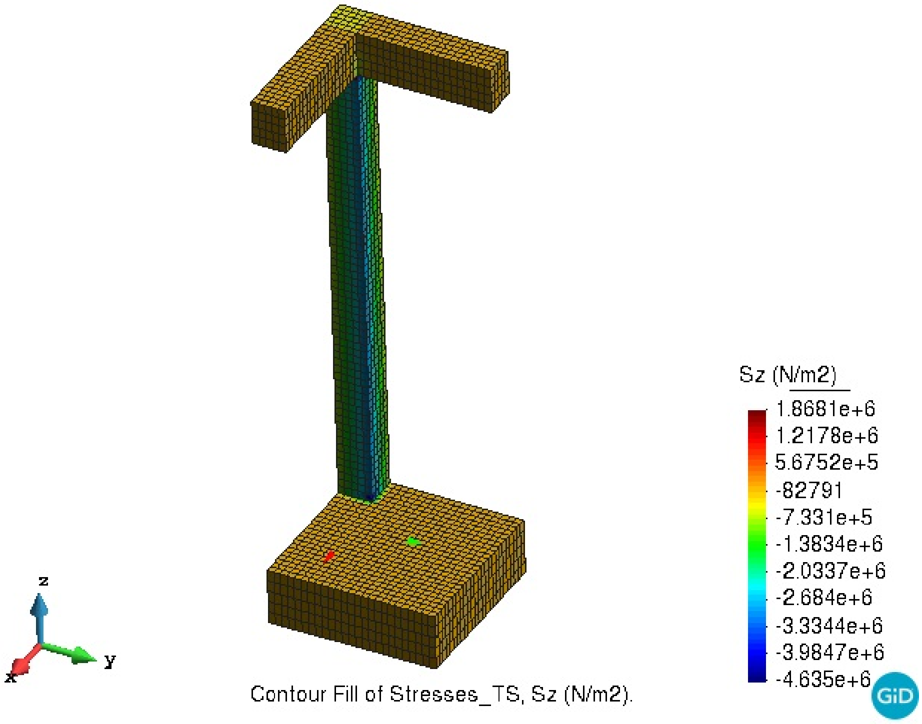


Figure 3.6: Normal stress in Z direction

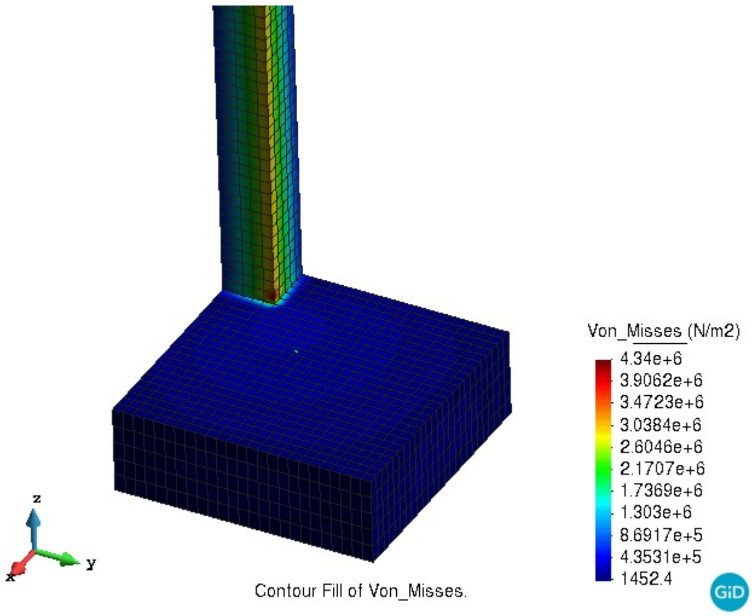


Figure 3.7: Von Misses stress