ASSIGNMENT 6.
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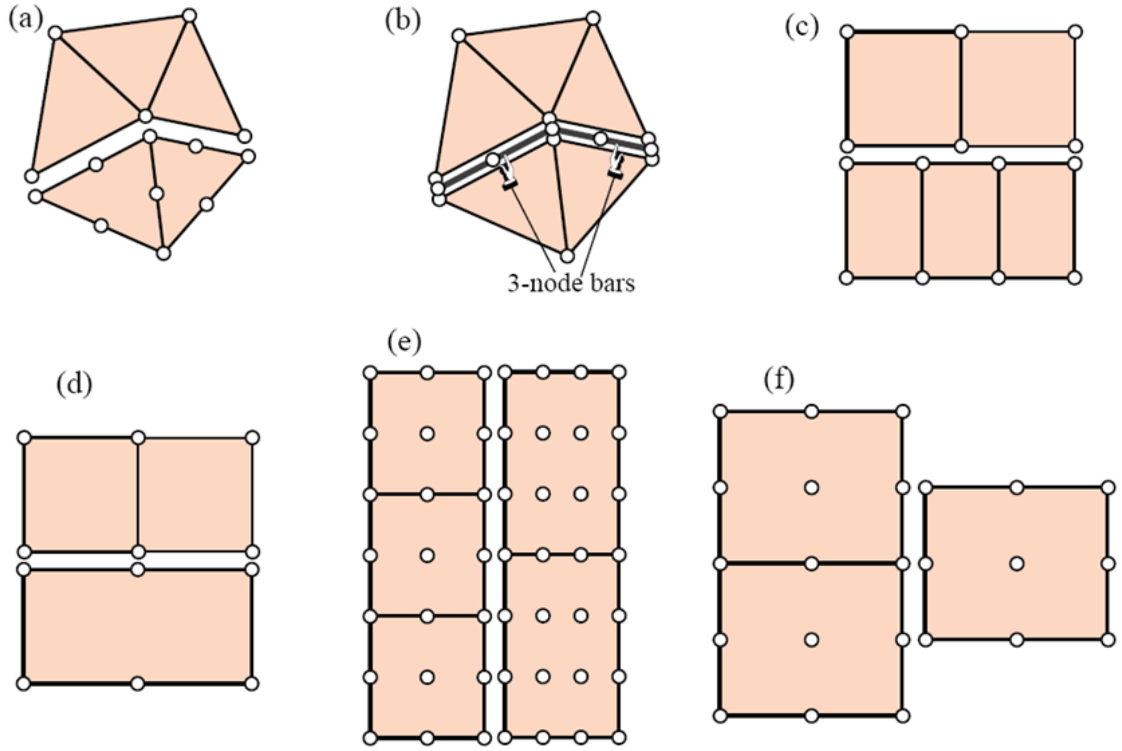
ASSIGNMENT 6.1



The before meshes fails interelement compatibility:

In (a),(b) and (c) the reason is because some nodes do not match.

(a) contains only one type of element but nodes does not match, but (b) and (c) mixes different element types and also some nodes does not match.

In (d), (e) and (f) nodes and DOFs match but some sides do not, leading to violations of C^0 continuity.

ASSIGNMENT 6.2

1 3 2

ξ=-1 ξ=0 ξ=1

$x_1 = 0$
$x_2 = L$
$x_3 = \frac{1}{2}$ L+ αL = L($\frac{1}{2}$ + α)

x= $x_1\ N_1^e(\xi) + x_2\ N_2^e(\xi) +\ x_3\ N_3^e(\xi)$ = $x_2\ N_2^e(\xi) +\ x_3\ N_3^e(\xi)$

$N_2^e(\xi) = \frac{1}{2}$ ξ (ξ +1)
$N_3^e(\xi) = 1 - \xi^2$

x= L$\frac{1}{2}$ ξ (ξ +1) + L($\frac{1}{2}$ + α) $(1 - \xi^2)$

x= L(α$(1 - \xi^2)$ +$\frac{1}{2}$ (ξ +1))

J= $\frac{dx}{d\xi}$= $\frac{1}{2}$ L - 2α ξL

α = $\frac{1}{4\xi}$

if ξ=-1 α = - $\frac{1}{4}$
and if ξ=1 α =+ $\frac{1}{4}$
α =+ $\frac{1}{4}$ *and* α =- $\frac{1}{4}$ are the values which makes J=0

Now we will compute the axial strain:
B=$\frac{dN}{dx} = J^{-1}\frac{dN}{d\xi}$

N=[$N_1\ N_2\ N_3$] = [$\frac{1}{2}$ ξ (ξ -1) , $\frac{1}{2}$ ξ (ξ +1) ,$(1 - \xi^2)$]

J= $\frac{1}{2}$ L - 2α ξL = $\frac{1-4\alpha\,\xi}{2}$ L,
$J^{-1} = \frac{2}{L(1-4\alpha\,\xi)}$

B=$\frac{2}{L(1-4\alpha\,\xi)}$ [ξ - $\frac{1}{2}$, ξ + $\frac{1}{2}$, -2 ξ]

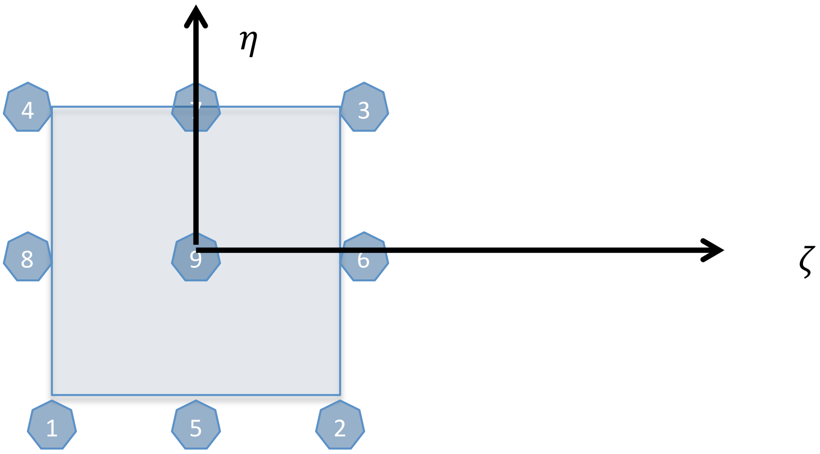
The end points are ξ=-1 and ξ=1:
B(ξ=1)= $\frac{2}{L(1-4\alpha)}$ [$\frac{1}{2}$, $\frac{3}{2}$, -2]
B(ξ=-1)= $\frac{2}{L(1+4\alpha)}$ [$-\frac{3}{2}$, -$\frac{1}{2}$, 2]

if α =+ $\frac{1}{4}$ B(ξ=1) becomes infinite, and if α =- $\frac{1}{4}$ B(ξ=-1) also becomes infinite.

So the minimum α's for which J vanishes makes axial strain becomes infinite (that is a singularity) at the end points.

(We notice that B(ξ=0)= $\frac{1}{L}$ $[-1$, 1, 0] does not depend on α)

ASSIGNMENT 6.3



We have a 9-node plane stress element, which it is initially a perfect square. In the before figure we can see the 9 nodes.

The 9 shape functions are now:

$N_1^e = \frac{1}{4}(\xi - 1)(\eta - 1)\xi\eta$
$N_2^e = \frac{1}{4}(1 + \xi)(\eta - 1)\xi\eta$
$N_3^e = \frac{1}{4}(1 + \xi)(1 + \eta)\xi\eta$
$N_4^e = \frac{1}{4}(\xi - 1)(1 + \eta)\xi\eta$
$N_5^e = \frac{1}{2}(1 - \xi^2)(\eta - 1)\eta$
$N_6^e = \frac{1}{2}(1 + \xi)(1 - \eta^2)\xi$
$N_7^e = \frac{1}{2}(1 - \xi^2)(1 + \eta)\eta$
$N_8^e = \frac{1}{2}(\xi - 1)(1 - \eta^2)\xi$
$N_9^e = (1 - \xi^2)(1 - \eta^2)$

We suppose L the mid side of the square.
x= $L(-N_1^e - N_8^e - N_4^e + N_3^e + N_6^e + N_2^e + \alpha N_5^e)$
y = $L(-N_1^e - N_5^e - N_2^e + N_4^e + N_7^e + N_3^e)$

$0 \leq \alpha \leq 1$, when $\alpha = 0$ node 5 is in the initial position, and while α approach to 1 node 5 is getting close to node 2.
We want to know at which α value Jacobian at node 2 vanishes. (This result is important in the construction of “singular elements” for fracture mechanics).

$$\mathbf{J} = \frac{\partial(x,y)}{\partial(\xi,\eta)} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

and

x = L($\frac{1}{4}$ (ξ - 1) $(\eta - 1)\xi\,\eta$ +…) gives us:

x= $\frac{L}{2}$(- η^2ξ + $\eta^2\xi^2$ $(1 - \alpha) + 2$ ξ + $\alpha(\xi^2\eta - \eta$ +η^2))

And operating similarly we obtain:
y= $\frac{L}{2}$(2 $\xi^2\eta$ + (1 - ξ^2) $(\eta^2 + 2\eta + 1)$)

J= $\frac{L}{2}\begin{bmatrix} -\eta^2 + 2\eta^2\xi \quad (1 - \alpha) + 2 + 2\alpha\xi \quad \eta & -2\xi \quad (\eta^2 + 1) \\ 2\xi\,\eta\big(-1 + \xi(1 - \alpha)\big) + \alpha(\xi^2\eta + 2\eta - 1) & 2\,(\eta(1 - \xi^2) + 1) \end{bmatrix}$

We substitute the ξ $and\ \eta$ values for node 2:
ξ$= 1$, $\eta = -1$

J= $\frac{L}{2}\begin{bmatrix} 3 - 4\alpha & -4 \\ 0 & 2 \end{bmatrix}$ = L$(3 - 4\alpha)$

J=0 when $\alpha = \frac{3}{4}$ =0,75

So, when node 5 is at just 0,25 L of node 2 Jacobian determinant at 2 vanishes.
In this case node 5 coordinates would be:
ξ =0,75L
η = -L

ASSIGNMENT 6.4

A stiffness matrix that has the correct rank (proper rank) is called rank sufficient and by extension, the associated element.

Rank sufficiency for numerically integrated finite elements can be computated as follows:

r = $\min(n_F - n_R, n_E n_G)$
d=$(n_F - n_R) - r$

r is the rank of K^e
n_G = number of Gauss points (in the intrgration)
n_E is the order of the stress-matrix E
n_F is the number of degrees of freedom
n_R is the number of independent rigid body modes

To attain rank sufficiency $n_E n_G \geq n_F - n_R$, $n_G \geq \frac{n_F - n_R}{n_E}$

If we have an hexahedrom $n_F = 3n$, where n is the number of nodes

$$n_R = 8$$
$$n_E = 3$$

$n_G \geq \frac{n_F - n_R}{n_E} = \frac{3n-8}{3}$

And the results are in the next table:

Element	n	n_F	$\boldsymbol{n_G}$
8-node hexahedron	8	24	**6**
20-node hexahedron	20	60	**18**
27-node hexahedron	27	81	**25**
64-node hexahedron	64	192	**62**