Computational Structural Mechanics and Dynamics

Assignment – Plates

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Assignment A

Analyse the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate, using a 5x5 mesh.

Like with beams, working with plates means choose between different theories that differ for the rotation of the normal to the middle plane.
On one hand, the classic thin plate theory based on the assumptions formalized by Kirchhoff that establishes that the normal remains straight and orthogonal to the middle plane after any deformation. This theory is restricted for those situations in which $\frac{thinkness}{average\ side} \leq 0.1$ and the elements needs C^1 continuity, that can produce some issues for deriving and conforming deflection field.
On the other hand, one of the other more famous theories, called Reissner-Mindlin. This theory says that the normal to the plate do not remains perpendicular to the mind-plane after deformation, allowing the effects of transverse shear deformation. R-M theory allows the user to use C^0 continuity elements, but other difficulties might appear for thin plate situations as the shear locking defect.

<u>The shear locking defect for R-M elements</u>
An additional shear stress is introduced in the element, which actually does not occur in the plate, causing the element to reach equilibrium with smallest displacements. The element appears to be stiffer that it actually is and the computed bending displacements smaller than they should be.
This effect can be observed working with the stiffness matrices for a isotropic plate of constant thickness under nodal point loads. The global equilibrium can be written as

$$(K_b + K_s)a = f$$

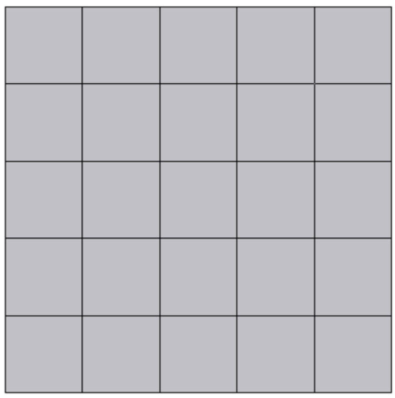
where K_b and K_s are the bending and shear stiffness contributions for a element.
After some algebra, the equation remains as

$$\left(\bar{K}_b + \frac{1}{\beta}\bar{K}_s\right)a = O(a_k) \qquad \beta = \frac{Et^2}{12(1-\nu^2)G}$$

Looking to the previous equations, it is clear that as β goes to zero $(t \rightarrow 0)$ the transverse shear dominates the solution and the influence of K_b is reduced, as almost negligible for very thing plates.

Then, plates with different thickness and a 25x25 mesh will be computed using MZC and R-M elements with the help of GID and Matlab software and results will be compared, in order to achieve some conclusions and check if the results obtained match with the theory.

<u>PROBLEM DESCRIPTION</u>



$E = 10.92\ N/m^2$
$\nu = 0.3$
$t = 0.001 - 0.01 - 0.02 - 0.1 - 0.4\ m$
$L = 1\ m$
Uniform load $Q = 1\ N/m$
Simple Support in each side of the square, so that

$$\begin{cases} For\ x - direction\ sides\ w = \theta_x = 0 \\ For\ y - direction\ sides\ w = \theta_y = 0 \end{cases}$$

DISPLACMENTS – RESULTS AND CONCLUSIONS

Maximum displacements obtained for each case are showed. As the differences can't be well observed in the graphic, the values are attached in a table being easier to check that the differences are higher that it seems at first sight.

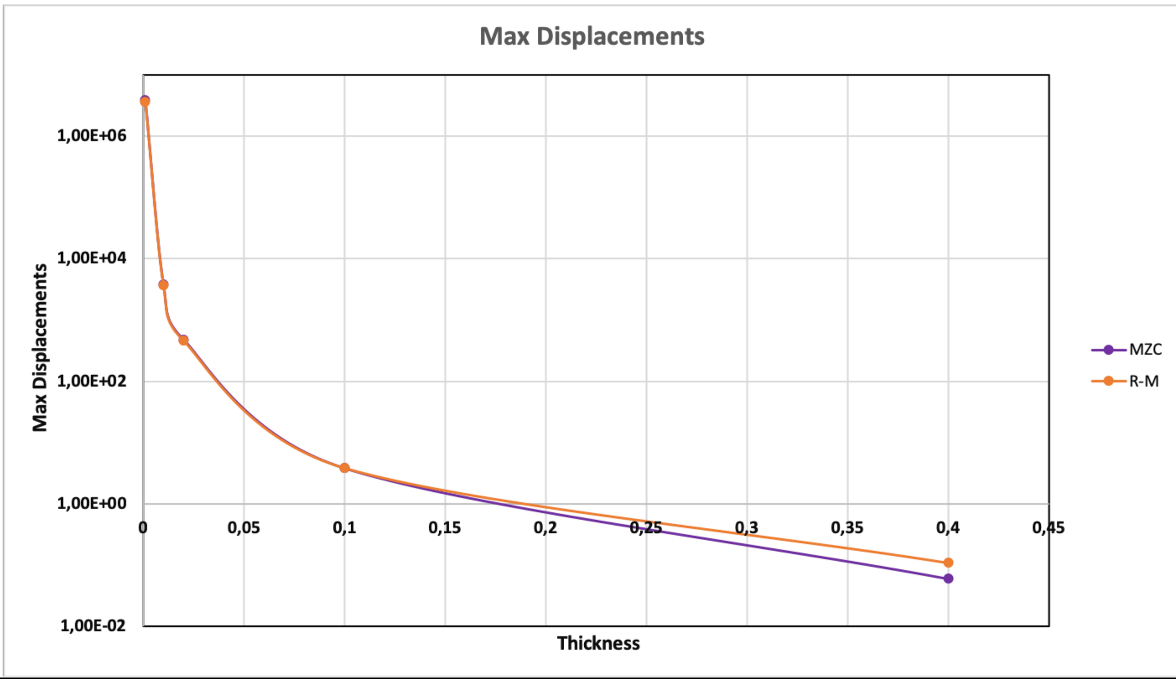


Figure 1. Max displacement graphic

Thickness	Max Disp MZC	Max Disp RM	MZC – RM
0,001	3860480,00	3641690,00	218790,00
0,01	3860,48	3643,77	216,71
0,02	482,56	456,26	26,30
0,1	3,86	3,85	0,01
0,4	0,06	0,11	-0,05

Table 1. Maximum displacement for each thickness

From $t = 0{,}001$ to $t = 0{,}1$ maximum displacements present differences, being always higher MZC solution. Due to the shear locking defect, the RM elements seems to be stiffer than they actually are, thus under the same uniform load Q the plate computed using RM elements should present lower maximum displacements.

At $t = 0{,}1$ in which shear locking defect is acting on RM results given lower values than it is expected, the resemblance between results suggest that MZC performance is already worse than form previous cases. The limit that restrict a good performance for classical thin plate theory has been reached, obtaining for $t \geq 0{,}1$ more accurate results for RM elements (look at $t = 0{,}4$). If a higher length is implemented $(L \geq 4\ m)$, results will behaviour correctly with any of the studied thickness for MZC elements (Table 2).

So that, it can be concluded that both models work as it expected. For so thin plates. MZC elements performs better becoming closer the results as t grows, reducing the shear locking defect for RM solutions.

Thickness	Displac MZC	Displac RM	MZC - RM
0,4	3,77E+01	3,69E+01	0,82

Table 2. Displacements for L=4m and t=0,4

Assignment B

Define and verify a patch test mesh for the MCZ element. Discuss the observed results.

Patch test is an indicator of the quality of a mesh. Doing a subsequent refinement of mesh, if the numerical solutions approaches towards exact solution, the test is said to be satisfied.
Working with plate elements three modalities of Patch test can be implement. For this case, the displacements of the nodes at the boundaries will be prescribed, then the displacement in the remaining nodes is computed and compared with the exact one. If both values are the same, the implemented mesh satisfied the test.
In order to verify the good representation of rigid body displacements a simple Patch test can be done with plate elements using quadrilateral linear elements, imposing the following displacement at the boundary nodes

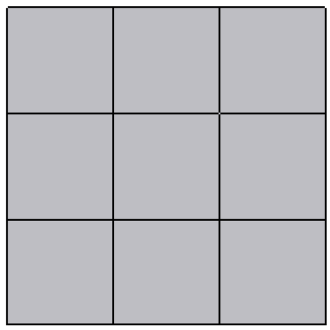
$$u = c - ax - by$$

where a, b and c are arbitrary numbers.

PROBLEM DESCRIPTION

A thin plate problem is going to be solved using 6 quadrilateral linear elements.
$L = 3\ m$
$a = b = c = 1$
MZC elements
Displacement u of nodes at each side of the square is computed using

$$u = 1 - x - y$$



and implement in the code as Boundary Conditions.
After solving the problem, the main point is comparing if the displacement at remaining nodes matches with the exact value, obtained substituting each node coordinates in the previous equation.

Node coord	Num Solution	Exact solution
1, 1	-1	-1
1, 2	-2	-2
2, 1	-2	-2
2, 2	-3	-3

It can be seen that for the selected mesh the Patch test is satisfied. It is true that for those plate elements that satisfies C^1 continuity, being compatible with Kirchhoff theory the patch test is not really needed. In this case it will be useful for verify that the code doesn't have any mistake, whereas in order cases it will be important to guarantee the convergence condition.
It is important to add that, using with MZC another kind of element instead of regular rectangles this test will not work property.