1)

In this example, a plate simply supported plate with uniform load is to be analyzed. The following are the material properties:

E = 10.92N/m^2 and nu = 0.3

The plate is under a uniform load of 1 N/m. The domain is a 5*5 square plate to be discretized using quadrilateral elements. Different types of elements are to be used in order to study the shear locking effect. These namely are the MZC element and the Reissner Mindlin (RM) element. In order to conduct this analysis, different thicknesses are used of descending magnitude.

A typical contour plot for the displacement in the z-direction is shown in Figure 1. This shows that the maximum displacement occurs at the center of the plate while the sides are zero due to the fixed boundary condition of simply supported plate.

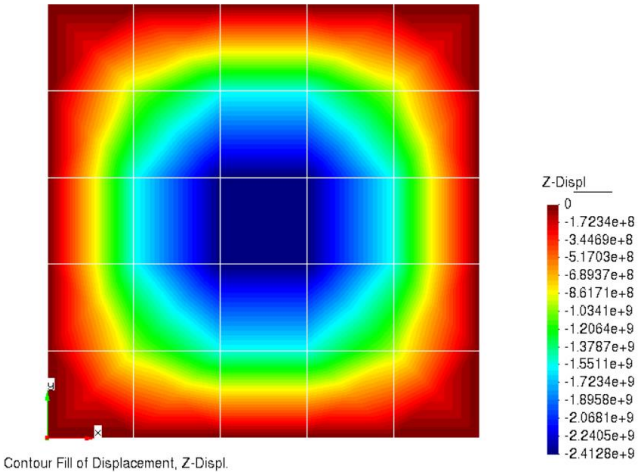

Figure 1 Z-displacement contour plot

The results are shown in Table 1.

Thickness	MZC z-disp.	RM z-disp.	Difference %
0.001	2.41E+09	2.28E+09	5.70E+00
0.01	2.41E+06	2.28E+06	5.70E+00
0.02	3.02E+05	2.85E+05	5.96E+00
0.1	2.41E+03	2.28E+03	5.70E+00
0.4	3.77E+01	3.69E+01	2.17E+00

Table 1 z-displacement for different thicknesses

The results for the log of the displacement in the z-direction and the thickness is shown in Figure 2.

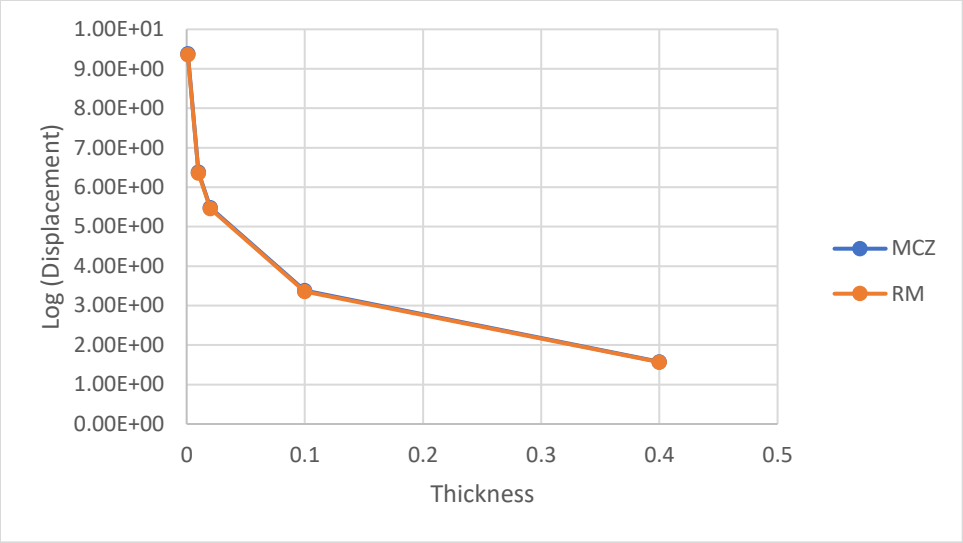

Figure 2 Log (z-displacement) vs. thickness

The results show that the values of the displacement obtained for both type of elements are quite similar thus in order to better compare the results the percentage difference between the displacement in the z-direction of both elements and the thickness is plotted in Figure 3.

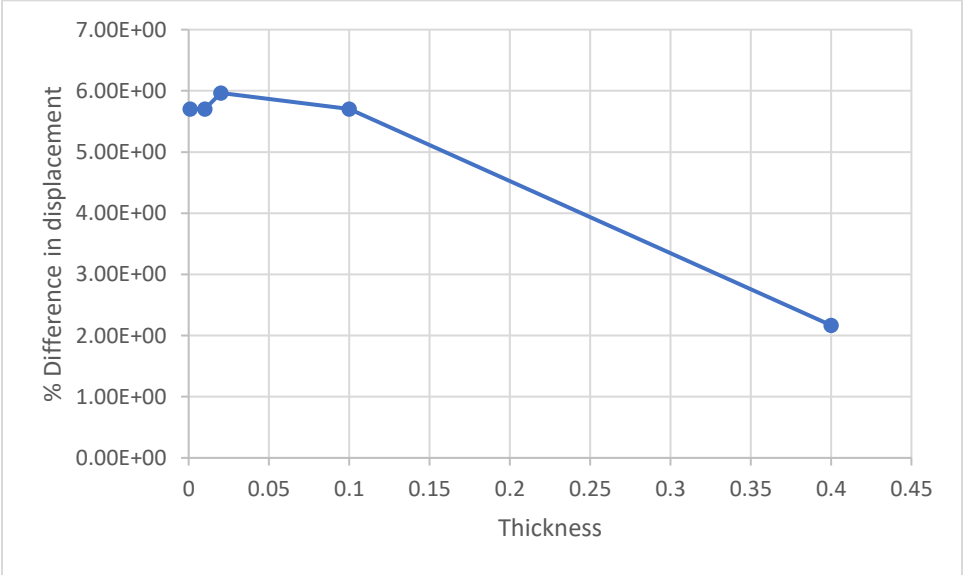

Figure 3 % difference (z-displacement) vs. thickness

It could be seen from the results that the RM element yields values for the z-displacement that are smaller compared to those obtained using the MZC element. It could be also noted that this difference is reduced as the thickness of the plate increases. The difference between the elements is highest when the thickness of the plates is small. That's due to the shear locking effect induced in the RM element. The thickness of the plate is the main

factor governing the shear locking effect. As the thickness gets smaller and approaches the stiffness of the element increases to infinity. The opposite is true, as the thickness of the plate increases, the shear locking effect vanishes thus justifying the decrease in the difference between the results obtain using both types of elements.

2)

In order to conduct the patch test for MCZ element, a 3*3 plate is meshed using rectangular elements (Figure 4) and then using arbitrary quadrilateral elements (Figure 5). The first step in applying the patch test is to assume a linear displacement field, uz = 1 - x – y, in the plate and a solution is obtained for the nodal displacement. This is followed by applying a linear boundary condition, if the solution obtained for the central elements satisfies the results obtained from the previous step, then the elements converge, otherwise, convergence is not assured.

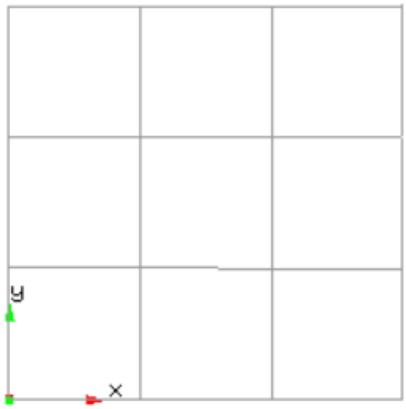

Figure 4 Rectangular elements mesh

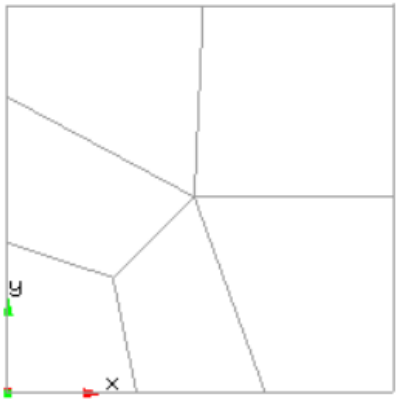

Figure 5 Arbitrary quadrilateral elements

Nodes	Exact solution	Patch test solution
(1,1)	-1	-1
(2,1)	-2	-2
(1,2)	-2	-2
(2,2)	-3	-3

Table 2 Results for rectangular elements mesh

The results of the structured mesh are identical to the results obtained from the assumed displacement filed (Table 2). Thus, the MCZ element pass the patch test assuring its convergence.

Node	Exact solution	Patch test solution
(0.8543, 1.8745)	-1.967978	-1.84679
(0.6077, 0.9728)	-0.748758	-0.67316
(2.1058, 1.9695)	-2.593971	-2.92436
(1.3048, 0.8799)	-1.534727	-1.7953
(2.4986, 1.3639)	-2.894686	-2.2745
(2.9783, 0.4682)	-2.339489	-2.7355

Table 3 Results for arbitrary quadrilateral elements mesh

In order to obtain the results of the quadrilateral elements mesh, Ramseries plate solver was used. The results of the arbitrary quadrilateral element (Table 3) mesh are not equivalent to the values of the assumed displacement field. Thus, the MCZ element does not pass the patch test and convergence is not assured in the case of arbitrary MCZ quadrilateral elements.