PRACTICE 3

EXERCISE 1: Clamped plate with a uniform load

Firstly, the analytical solution will be deduced. The analytical solution is calculated using a structural handbook. The deflection in the center of the plate (W_{max}) is,

$$W_{max} = h \cdot \delta \cdot \frac{q}{E}\left(\frac{c}{h}\right)^4 = 0.1m \cdot 0.015 \frac{10^4}{3 \cdot 10^4}\left(\frac{4}{0.1}\right)^4 = 1.28 \cdot 10^{-3}\ m$$

$$\delta = \frac{1-\nu^2}{\delta_0} = \frac{1-0.2^2}{64} = 0.015$$

$$\delta_0 = 32(1+\alpha^4) = 32\left(1+\left(\frac{4}{4}\right)^2\right) = 64$$

$$W_{max} = 1.28 \cdot 10^{-3}\ m < \frac{h}{2} = 0.05\ m \Longrightarrow \text{Small displacement}$$

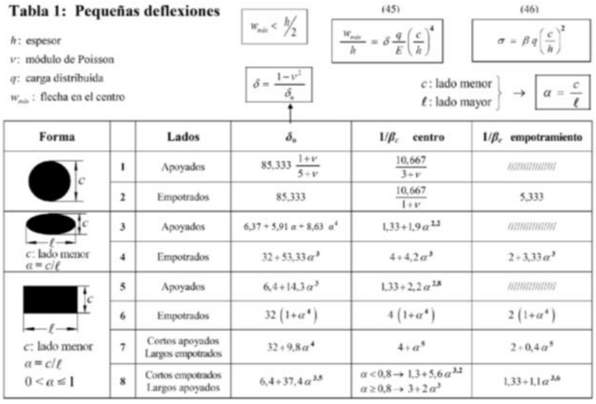


Tabla 1: Pequeñas deflexiones

h: espesor
ν: módulo de Poisson
q: carga distribuida
$w_{máx}$: flecha en el centro

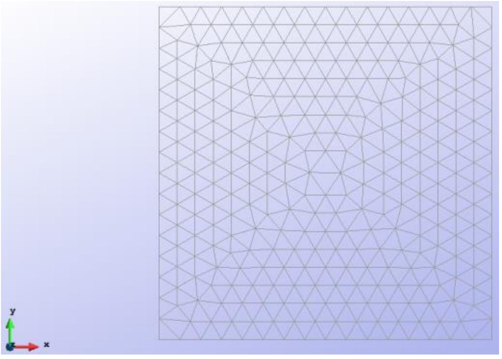
$$w_{máx} < h/2 \qquad \frac{w_{máx}}{h} = \delta \frac{q}{E}\left(\frac{c}{h}\right)^4 \ (45) \qquad \sigma = \beta q \left(\frac{c}{h}\right)^2 \ (46)$$

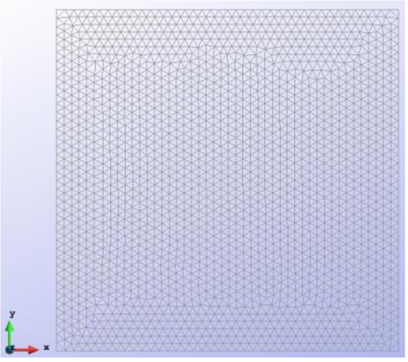
$$\delta = \frac{1-\nu^2}{\delta_o}$$

c: lado menor, ℓ: lado mayor $\rightarrow$ $\alpha = \frac{c}{\ell}$

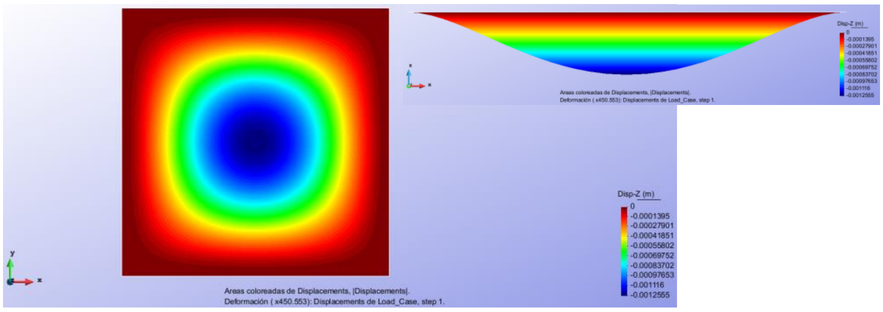
Forma		Lados	δ_o	$1/\beta_c$ centro	$1/\beta_e$ empotramiento
c	1	Apoyados	$85{,}333\,\frac{1+\nu}{5+\nu}$	$\frac{10{,}667}{3+\nu}$	▨
	2	Empotrados	85,333	$\frac{10{,}667}{1+\nu}$	5,333
c: lado menor $\alpha = c/\ell$	3	Apoyados	$6{,}37+5{,}91\,\alpha+8{,}63\,\alpha^4$	$1{,}33+1{,}9\alpha^{2,2}$	▨
	4	Empotrados	$32+53{,}33\alpha^3$	$4+4{,}2\alpha^3$	$2+3{,}33\alpha^3$
c: lado menor $\alpha = c/\ell$ $0<\alpha\le 1$	5	Apoyados	$6{,}4+14{,}3\alpha^3$	$1{,}33+2{,}2\alpha^{2,8}$	▨
	6	Empotrados	$32\left(1+\alpha^4\right)$	$4\left(1+\alpha^4\right)$	$2\left(1+\alpha^4\right)$
	7	Cortos apoyados Largos empotrados	$32+9{,}8\alpha^4$	$4+\alpha^5$	$2+0{,}4\alpha^5$
	8	Cortos empotrados Largos apoyados	$6{,}4+37{,}4\alpha^{3,5}$	$\alpha<0{,}8 \rightarrow 1{,}3+5{,}6\alpha^{3,2}$ $\alpha\ge 0{,}8 \rightarrow 3+2\alpha^3$	$1{,}33+1{,}1\alpha^{3,6}$

The results obtained by the software for triangular elements should be very close to the analytical solution. This comparison can be used to validate the input data. The displacement at the middle is 0.0012555 m, which is very similar to the analytical. Therefore, the error is around 2% for 0.25-size elements. In case the size of the element is decreased, the error will decrease. For example, the displacement is 0.0012564 using 0.1-size elements, where the error is 1.85%.

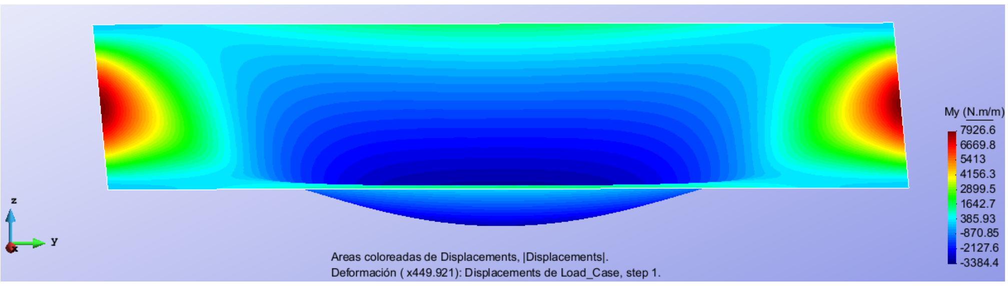




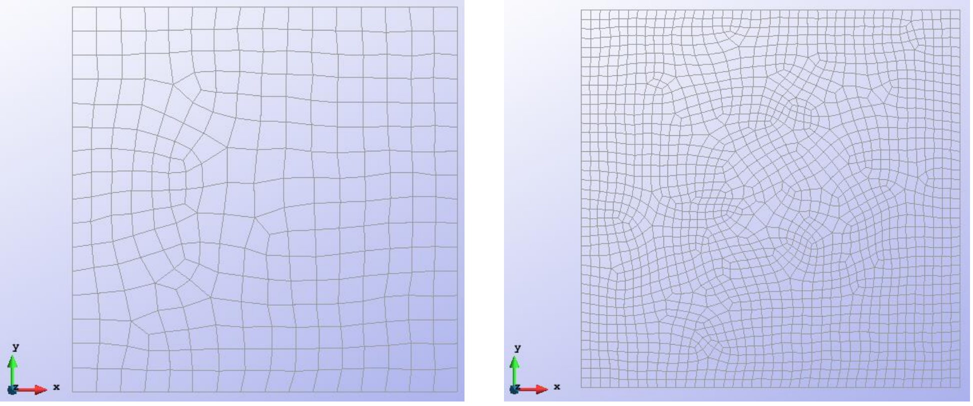
The greatest displacement are in the center of the plate and in the borders, due to the fixed size, the displacement are restringed. This boundary condition does not allow any displacement or rotation in the boundary because it is clamped.



As regards the bending moment, near the edge the bending moment is positive and in the middle part the sign is different. The sign modification is because the plate is clamped in the bounday. If it had been simply supported, the bending moment would have the same sign in whole the plate. The change of the sign also changes the stresses in the faces of the section. This aspect is import ,for example, when the reinforment of a concrete plate is being designed because the longitudinal bars are located in the sides where the tensile stresses are applied. Hence the longitudinal should be changed of side.

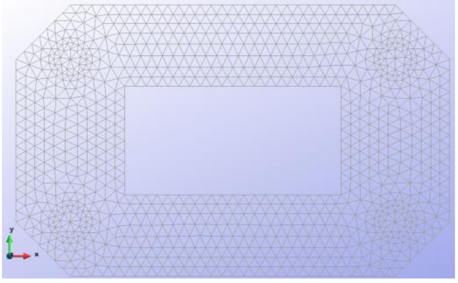


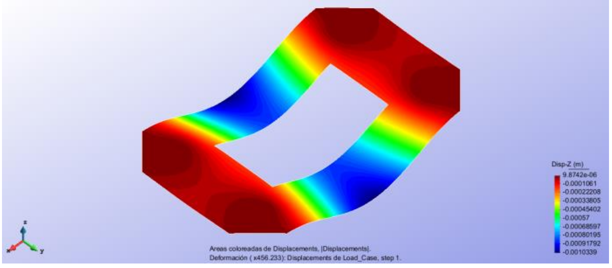
The same plate has been implemented using quadrilateral elements. The stress and displacements pattern are very similar. The displacement in the center of the plate is 0.0012258 m when 0.25-size quadrilateral elements are used and the error is 4.1%. However, the displacement is 0.0012535 m and the error 2.1% for 0.1-size elements.



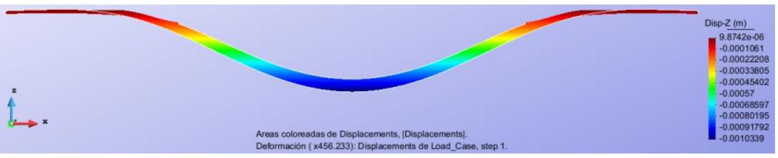
EXERCISE 2: Thin plate with interval hole

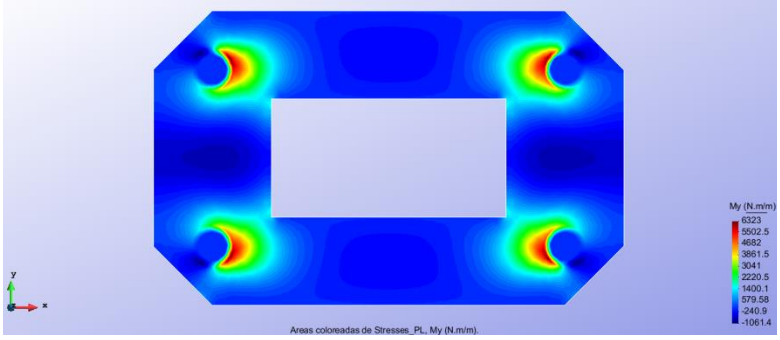
The plate has a hole in the middle and it is supported by four circular piles located in the extremes. The uniform distributed load push down the plate and the main deformation will be located in the center part of longest edge.





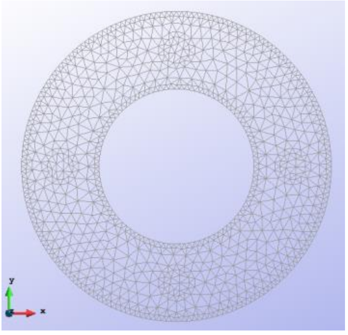
The maximum bending moments are near the piles and the negative ones in the middle part of the plate. The profile view shows clearly the change in the sign of the moment and where are located. The displacements are minimum in the extremes of the plate where the piles are located.

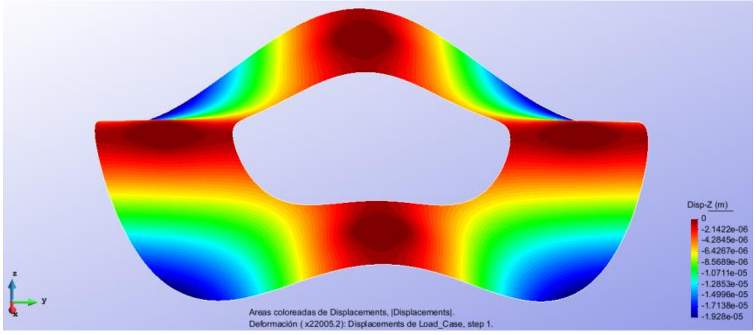




EXERCISE 3: Thick circular plate with internal hole

The circular plate is meshed using triangular elements and pushed down by a uniform loading. The plate between the piles is displaced down and the parts of the plate over the piles are not subjected to any displacement.





The loading will also induce shear forces, especially near the piles. The plate is displaced down and this movement will tend to drag the piles horizontally. As regards bending moments, the maximum ones will be located near the piles. These bending moments will compress the down-side of the plate and tensile forces are applied in the top-side of the plate. Furthermore, the bending moments have opposite direction between the piles, where the tensile reinforcement will have to change of side.