Homework 1

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Exercise 1: Thin plate under dead weight

The problem has been solved using 3-node triangles and 4-node quadrilaterals with different element sizes: 1, 0.5, 0.25 and 0.125; and 6-node triangles, 8-node quadrilaterals and 9-node quadrilaterals with element sizes 1, 0.5 and 0.25. The results are compared with those obtained with a mesh composed of 9-node quadrilaterals with element size 0.125: A vertical displacement at point B equal to $-2.3037 \cdot 10^{-6} m$ and y-stress equal to $2.50780 \cdot 10^{5} Pa$ at the center of side ED. Figures 1 and 2 show the convergence plot obtained with the different elements when calculating the relative error with respect to the solution obtained with the finest mesh of 9-node quadrilaterals. The convergence plot for the displacement (figure 1) shows a convergence rate much slower for linear elements than the quadratic ones. Amongst the quadratic elements, the error obtained is similar for all the elements, however, the linear quadrilateral element is better than the 3-node triangle. The results obtained for the stresses (Fig. 2) are worst, especially for linear elements since they depend on the derivative of the shape functions. It can be concluded that the best results are those obtained with the 8-node and 9-node quadrilateral elements.

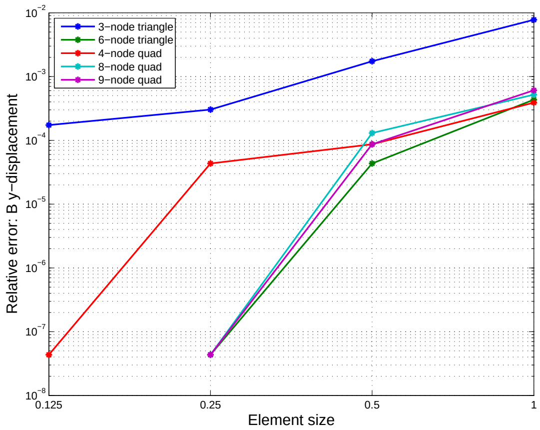


Figure 1: Relative error of y-displacement at point B vs element size

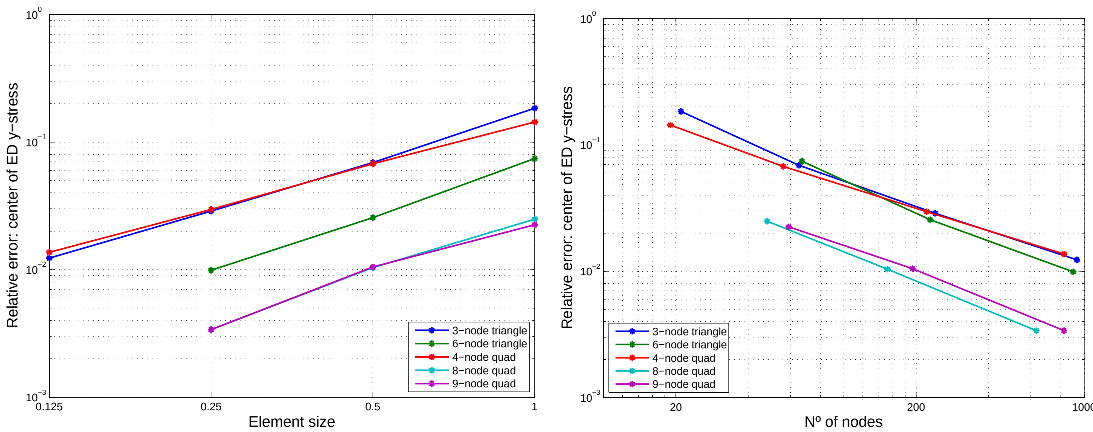
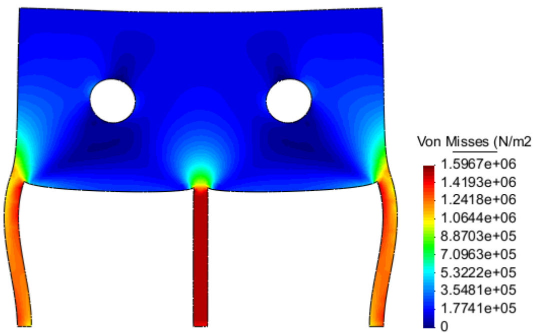


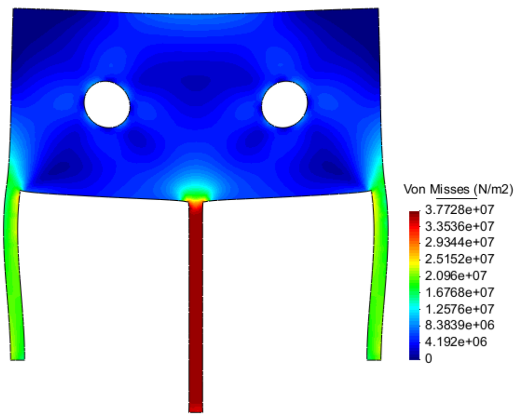
Figure 2: Relative error of y-stress at the center of segment ED vs element size and number of nodes

Exercise 2: Plate with two holes

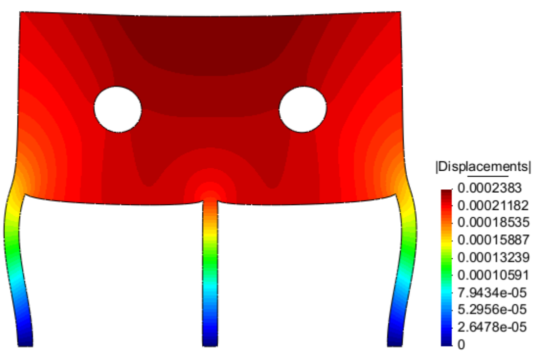
The problem has been solved using 3-node triangles with an element size equal to 0.05 (4233 nodes). The problem has been solved first fixing all the columns (figure 3a) and then for a displacement $\delta = -0.01$ m in the central column (figure 3b). The results obtained show that the maximum Von Mises stresses for $\delta = -0.01$ m are much larger ($\sigma_{VM} = 3.7728 \cdot 10^7$ Pa) than for $\delta = 0$ ($\sigma_{VM} = 1.5967 \cdot 10^6$ Pa) .



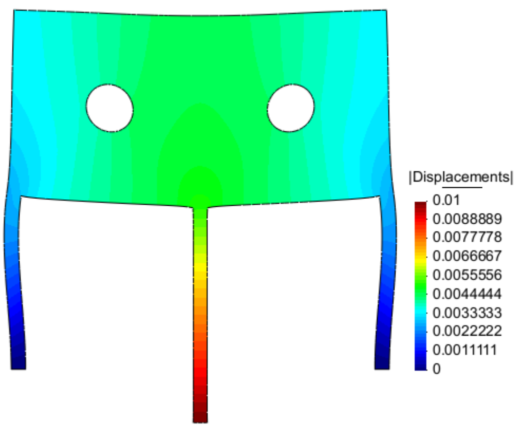
(a) Von Misses stresses for $\delta = 0$



(b) Von Misses stresses for $\delta = -0.01$ m



(a) Displacement field for $\delta = 0$



(b) Displacement field for $\delta = -0.01$ m

Exercise 3: Plate with ventilation hole

The problem has been solved using 4-node quadrilateral elements with an element size of 0.01 (34020 nodes). With this mesh, the problem is converged except for the concentration of stresses in the vertices of the inner hole. The distribution of Von Misses stresses obtained is depicted in figure 5. As can be seen, the stress field obtained for the concrete section is smoother than for the steel section. Moreover, in the steel section there are larger stresses because concentration of stresses appear. The displacements obtained are low, with a maximum displacement equal to 0.005404 at the middle of the plate.

(a) Von Mises stresses for the concrete and steel sections

(b) Von Mises stresses for the concrete alone

Figure 5: Von Mises stresses

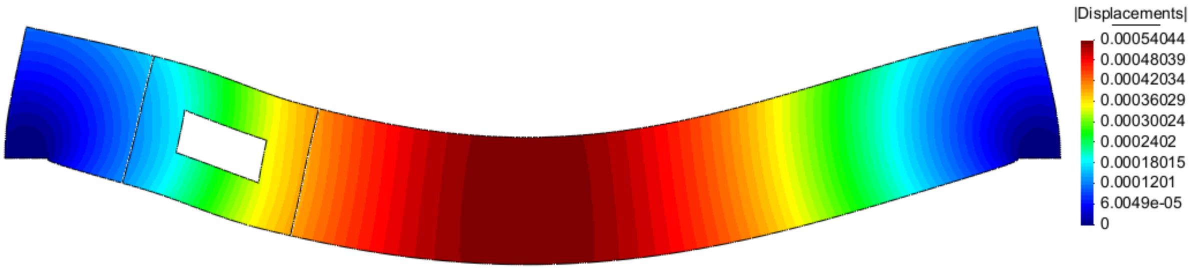


Figure 6: Magnitude of the displacements

Exercise 4: Prismatic water tank

The problem has been solved using 4-node quadrilaterals with an element size equal to 0.025 (4745 nodes). The results obtained show that the maximum Von Mises stresses are obtained in the inner vertice, where there is a concentration of stresses ($\sigma_{VM} = 9.1509 \cdot 10^5$ Pa), and the maximum displacements are obtained for the upper section of the damn. It would be useful to round the corners of the damn to avoid stress concentration.

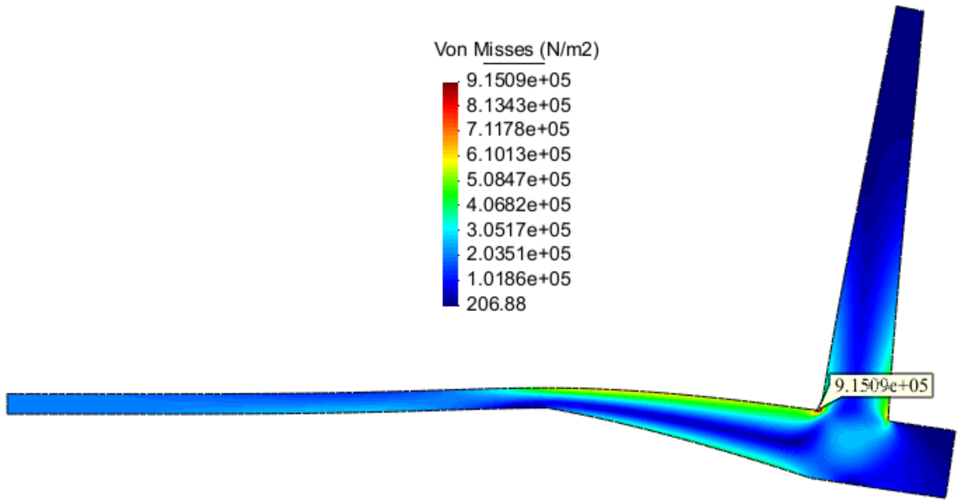


Figure 7: Von Misses stresses

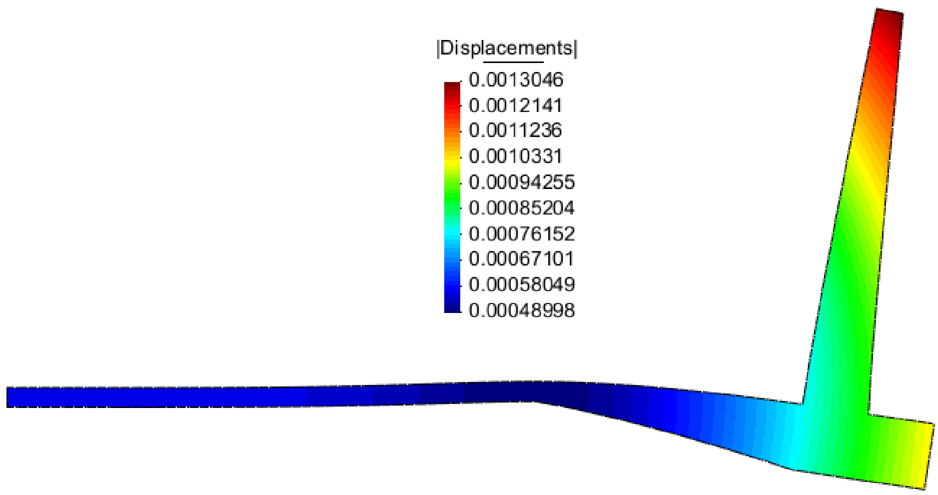


Figure 8: Module of the displacements