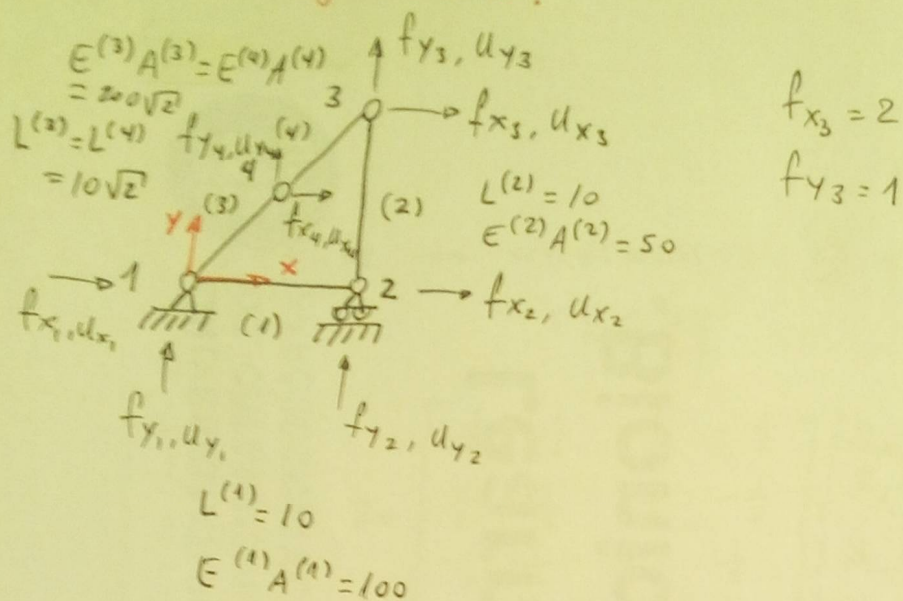
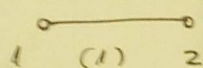


Assignment 2.



Globalized element stiffness

For member 1:

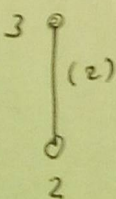


$$\varphi^{(1)} = 0^\circ \quad \begin{aligned} \cos 0^\circ &= 1 \\ \sin 0^\circ &= 0 \end{aligned}$$

$$\frac{EA}{L} = \frac{100}{10} = 10$$

$$\begin{bmatrix} f_{x1}^{(1)} \\ f_{y1}^{(1)} \\ f_{x2}^{(2)} \\ f_{y2}^{(1)} \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x1}^{(1)} \\ u_{y1}^{(1)} \\ u_{x2}^{(1)} \\ u_{y2}^{(1)} \end{bmatrix}$$

For member 2:

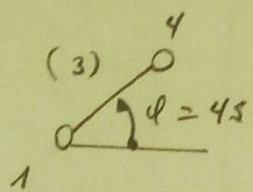


$$\varphi = 90^\circ \rightarrow \begin{aligned} \cos 90^\circ &= 0 \\ \sin 90^\circ &= 1 \end{aligned}$$

$$\frac{EA}{L} = \frac{50}{10} = 5$$

$$\begin{bmatrix} f_{x2}^{(2)} \\ f_{y2}^{(2)} \\ f_{x3}^{(2)} \\ f_{y3}^{(2)} \end{bmatrix} = 5 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x2}^{(2)} \\ u_{y2}^{(2)} \\ u_{x3}^{(2)} \\ u_{y3}^{(2)} \end{bmatrix}$$

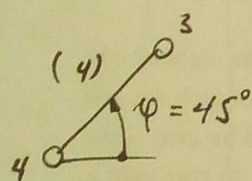
For member 3:



$\sin 45 = \cos 45 = \frac{\sqrt{2}}{2} = c = s$
 $\frac{EA}{L} = \frac{200\sqrt{2}}{10\sqrt{2}} = 20$
 $s^2 = c^2 = \frac{1}{2}$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = 20 \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

For member 4:



$\sin 45 = \cos 45 = c = s = \frac{\sqrt{2}}{2}$
 $\frac{EA}{L} = \frac{200\sqrt{2}}{10\sqrt{2}} = 20$
 $s^2 = c^2 = \frac{1}{2}$

$$\begin{bmatrix} f_{x_3}^{(4)} \\ f_{y_3}^{(4)} \\ f_{x_4}^{(4)} \\ f_{y_4}^{(4)} \end{bmatrix} = 20 \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_{x_3}^{(4)} \\ u_{y_3}^{(4)} \\ u_{x_4}^{(4)} \\ u_{y_4}^{(4)} \end{bmatrix}$$

Expanded Elements Stiffness Equations

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \\ f_{x_3}^{(1)} \\ f_{y_3}^{(1)} \\ f_{x_4}^{(1)} \\ f_{y_4}^{(1)} \end{bmatrix} = \begin{bmatrix} 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \\ u_{x_3}^{(1)} \\ u_{y_3}^{(1)} \\ u_{x_4}^{(1)} \\ u_{y_4}^{(1)} \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \\ f_{x_4}^{(2)} \\ f_{y_4}^{(2)} \end{bmatrix} = \begin{matrix} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{bmatrix} u_{x_1}^{(2)} \\ u_{y_1}^{(2)} \\ u_{x_2}^{(2)} \\ u_{y_2}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \\ u_{x_4}^{(2)} \\ u_{y_4}^{(2)} \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_2}^{(3)} \\ f_{y_2}^{(3)} \\ f_{x_3}^{(3)} \\ f_{y_3}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \begin{matrix} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \\ \begin{bmatrix} 10 & 10 & 0 & 0 & 0 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & 0 & 0 & -10 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & -10 & 0 & 0 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & 0 & 0 & 0 & 10 & 10 \end{bmatrix} \end{matrix} \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_2}^{(3)} \\ u_{y_2}^{(3)} \\ u_{x_3}^{(3)} \\ u_{y_3}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1}^{(4)} \\ f_{y_1}^{(4)} \\ f_{x_2}^{(4)} \\ f_{y_2}^{(4)} \\ f_{x_3}^{(4)} \\ f_{y_3}^{(4)} \\ f_{x_4}^{(4)} \\ f_{y_4}^{(4)} \end{bmatrix} = \begin{bmatrix} 10 & 10 & 0 & 0 & 0 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & 0 & 0 & -10 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & -10 & 0 & 0 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & 0 & 0 & 0 & 10 & 10 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(4)} \\ u_{y_1}^{(4)} \\ u_{x_2}^{(4)} \\ u_{y_2}^{(4)} \\ u_{x_3}^{(4)} \\ u_{y_3}^{(4)} \\ u_{x_4}^{(4)} \\ u_{y_4}^{(4)} \end{bmatrix}$$

$$\begin{array}{cccccccc} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{array} \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \end{bmatrix} \begin{bmatrix} u_{x_1} \rightarrow 0 \\ u_{y_1} \rightarrow 0 \\ u_{x_2} \\ u_{y_2} = \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} \begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \rightarrow 2 \\ f_{y_3} \rightarrow 1 \\ f_{x_4} \\ f_{y_4} \end{bmatrix}$$

these are multiples of each other.

The solution "blows up" because the coefficient matrix (master stiffness matrix) is singular.

The physical interpretation of singularity is that there are unsuppressed rigid body motions: the truss still floats in the (x, y) plane.