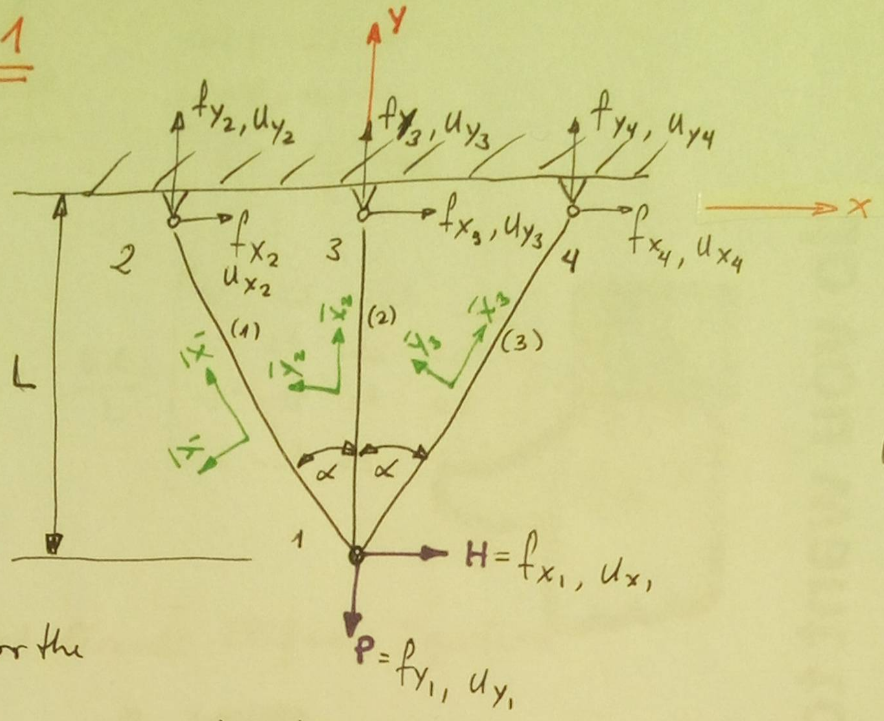
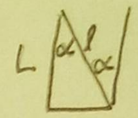
CSMD. Jorge Alvarez 12/02/17 1/.

Assignment 1



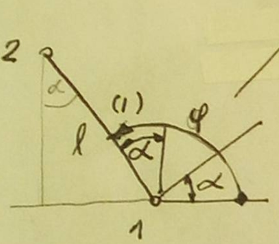


$\cos\alpha = \frac{L}{\ell}$

$\ell = \frac{L}{\cos\alpha}$

E, A the same for the 3 bars

a).



$\varphi = \alpha + 90$; $\alpha \neq 0$

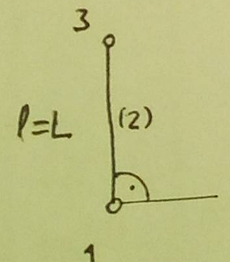
$\alpha > 90$

$\rightarrow$ c (−) $\rightarrow \cos(\underbrace{\alpha+90}_{\varphi}) = -\sin\alpha = -s$

$\rightarrow$ s (+) $\rightarrow \sin(\underbrace{\alpha+90}_{\varphi}) = \cos\alpha = c$

$$\frac{EA}{\ell} = \frac{EA\,c}{L}$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \frac{EAc}{L} \begin{bmatrix} s^2 & -sc & -s^2 & +sc \\ -sc & c^2 & +sc & -c^2 \\ -s^2 & +sc & s^2 & -sc \\ +sc & -c^2 & -sc & c^2 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}.$$

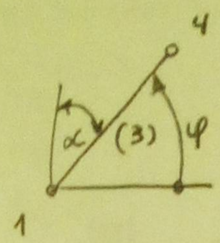


$\varphi = 90$ $\cos\varphi = 0$

$\sin\varphi = 1$

$$\frac{EA}{\ell} = \frac{EA}{L}$$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(2)} \\ u_{y_1}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$



$\varphi + \alpha = 90°$

$\cos\varphi = \sin\alpha$

$\sin\varphi = \cos\alpha$

$\frac{EA}{l} = \frac{EA}{L}\cos\alpha$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \frac{EA\,c}{L} \begin{bmatrix} s^2 & cs & -s^2 & -cs \\ cs & c^2 & -cs & -c^2 \\ -s^2 & -cs & s^2 & cs \\ -cs & -c^2 & cs & c^2 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

Expanded Elements Stifness Equations

For member 1:

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \\ f_{x_3}^{(1)} \\ f_{y_3}^{(1)} \\ f_{x_4}^{(1)} \\ f_{y_4}^{(1)} \end{bmatrix} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccccccc} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{array} \\ \begin{bmatrix} cs^2 & -sc^2 & -s^2c & +sc^2 & 0 & 0 & 0 & 0 \\ -sc^2 & c^3 & +sc^2 & -c^3 & 0 & 0 & 0 & 0 \\ -s^2c & +sc^2 & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ +sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \\ u_{x_3}^{(1)} \\ u_{y_3}^{(1)} \\ u_{x_4}^{(1)} \\ u_{y_4}^{(1)} \end{bmatrix}$$

For member 2:

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \\ f_{x_4}^{(2)} \\ f_{y_4}^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(2)} \\ u_{y_1}^{(2)} \\ u_{x_2}^{(2)} \\ u_{y_2}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \\ u_{x_4}^{(2)} \\ u_{y_4}^{(2)} \end{bmatrix}$$

For member 3:

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_2}^{(3)} \\ f_{y_2}^{(3)} \\ f_{x_3}^{(3)} \\ f_{y_3}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \frac{EA}{L} \begin{array}{c} \begin{array}{cccccccc} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{array} \\ \begin{bmatrix} s^2c & c^3 & 0 & 0 & 0 & 0 & -s^2c & -c^2s \\ c^2s & c^3 & 0 & 0 & 0 & 0 & -c^2s & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -s^2c & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_2}^{(3)} \\ u_{y_2}^{(3)} \\ u_{x_3}^{(3)} \\ u_{y_3}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

$$f = f^{(1)} + f^{(2)} + f^{(3)} = \left(K^{(1)} + K^{(2)} + K^{(3)}\right) u = K u$$

$$\frac{AE}{L} \begin{array}{c} \begin{array}{cccccccc} x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{array} \\ \begin{bmatrix} 2cs^2 & 0 & -s^2c & sc^2 & 0 & 0 & -s^2c & -c^2s \\ 0 & 2c^3+1 & sc^2 & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ sc^2 & -c^3 & -sc^2 & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -s^2c & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \end{array} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \\ f_{x_4} \\ f_{y_4} \end{bmatrix}$$

In the 5th row and column contain only zeros because in $u_{x_3} = 0$, there is not displacement and the system will be rotate around u_{x_3}, as a pivot point. Also $f_{x_3} = 0$

b).

BC:

Displacement:

$u_{x_2} = 0$

$u_{y_2} = 0$

$u_{x_3} = 0$

$u_{y_3} = 0$

$u_{x_4} = 0$

$u_{y_4} = 0$

Forces:

$f_{x_1} = H$

$f_{y_1} = -P$

Reduced Stiffness Equation

$$\begin{bmatrix} 2cs^2 & 0 \\ 0 & 2c^3+1 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}. \quad \blacksquare$$

c)

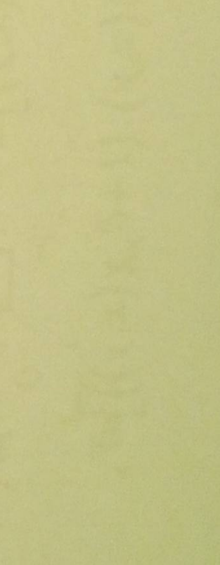
$$\left.\begin{aligned} 2cs^2 \cdot u_{x_1} &= H \\ (2c^3+1)\, u_{y_1} &= -P \end{aligned}\right\}$$

$$u_{x_1} = \frac{H}{2cs^2}$$

$$u_{y_1} = \frac{-P}{2c^3+1}$$

$\alpha \to 0$:

$$\lim_{\alpha \to 0} u_{x_1} = \frac{H}{2 \cdot \underbrace{\cos 0}_{\approx 1} \; \underbrace{\sin^2 0}_{\approx 0}} \approx \gg \cdots \infty$$

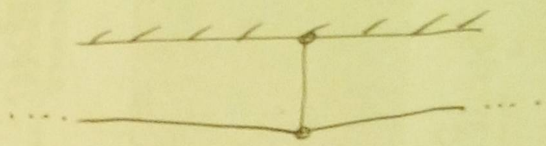


$$\lim_{\alpha \to 0} u_{y_1} = \frac{-P}{2\underbrace{\cos^3 0}_{1}+1} \approx -\frac{P}{3}$$

$\therefore$ U_{x1} will be bigger, that doesn't make physically sense

$\alpha \to \frac{\pi}{2}$.

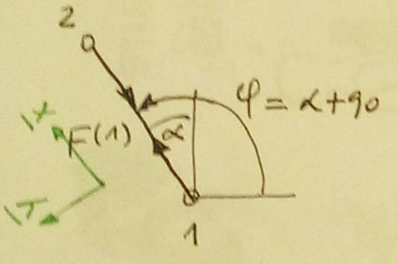
$$\lim_{\alpha \to \frac{\pi}{2}} \frac{H}{2\underbrace{\cos\frac{\pi}{2}}_{\approx 0}\cdot \underbrace{\sin^2\frac{\pi}{2}}_{\approx 1}} = \gg \ldots \infty$$



$$\lim_{\alpha \to \frac{\pi}{2}} \frac{-P}{2\underbrace{\cos^3\frac{\pi}{2}}_{\approx 0}+1} = -P$$

$\therefore$ u_{x_1} will be bigger. that doesn't make physically sense. $\blacksquare$

d)



$$\bar{u}^{(1)} = T^{(1)} u^{(1)}$$

$$u^{(1)} = \left[u_{x_1}\ u_{y_1}\ u_{x_2}\ u_{y_2}\right]^T$$

$\cos\varphi = -\sin\alpha = -s$
$\sin\varphi = \cos\alpha = c$

$$= \left[\frac{H}{2cs^2} \quad \frac{-P}{2c^3+1} \quad 0 \quad 0\right]$$

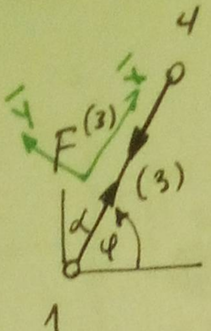
$$\begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_2} \\ \bar{u}_{y_2} \end{bmatrix} = \begin{bmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{bmatrix} \begin{bmatrix} \frac{H}{2cs^2} \\ -\frac{P}{2c^3+1} \\ 0 \\ 0 \end{bmatrix}$$

$\bar{u}_{x_2} = 0 \qquad \bar{u}_{x_1} = -\frac{H}{2cs} - \frac{Pc}{2c^3+1}$

$u_{y_2} = 0 \qquad u_{y_1} = -\frac{H}{2s^2} + \frac{Ps}{2c^3+1}$

$$d^{(1)} = \bar{u}_{x_2} - \bar{u}_{x_1} = 0 - \left(-\frac{H}{2cs} - \frac{Pc}{2c^3+1}\right) = \frac{H}{2cs} + \frac{Pc}{2c^3+1}$$

$$F^{(1)} = \frac{EA}{L} c\left(\frac{H}{2cs} + \frac{Pc}{2c^3+1}\right) = \frac{EA}{L}\left(\frac{H}{2s} + \frac{Pc^2}{2c^3+1}\right)$$



$$\cos\varphi = \sin\alpha = s$$
$$\sin\varphi = \cos\alpha = c$$

$$\frac{EA}{L}\cos\alpha$$

$$\bar{u}^{(3)} = T^{(3)} u^{(3)}$$

$$u^{(3)} = \left[u_{x_1}\ u_{y_1}\ u_{x_4}\ u_{y_4}\right]^T$$

$$= \left[\frac{H}{2cs^2}\ \ \frac{-P}{2c^3+1}\ \ 0\ \ 0\right].$$

$$\begin{bmatrix}\bar{u}_{x_1}\\ \bar{u}_{y_1}\\ \bar{u}_{x_4}\\ \bar{u}_{y_4}\end{bmatrix} = \begin{bmatrix} s & c & 0 & 0\\ -c & s & 0 & 0\\ 0 & 0 & s & c\\ 0 & 0 & -c & s\end{bmatrix}\begin{bmatrix}u_{x_1} \to \frac{H}{2cs^2}\\ u_{y_1} \to -\frac{P}{2c^3+1}\\ u_{x_4} \to 0\\ u_{y_4} \to 0\end{bmatrix}$$

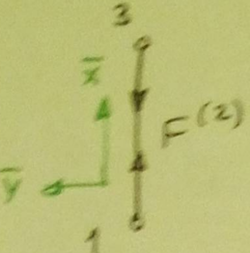
$$\bar{u}_{x_4} = 0 \qquad \bar{u}_{x_1} = \frac{H}{2cs} - \frac{Pc}{2c^3+1}$$
$$\bar{u}_{y_4} = 0$$
$$\bar{u}_{y_1} = -\frac{H}{2s^2} - \frac{Ps}{2c^3+1}$$

$$d^{(3)} = \bar{u}_{x_4} - \bar{u}_{x_1} = 0 - \left(\frac{H}{2cs} - \frac{Pc}{2c^3+1}\right) = \frac{Pc}{2c^3+1} - \frac{H}{2cs}$$

$$F^{(3)} = \frac{EA}{L}c\left(\frac{Pc}{2c^3+1} - \frac{H}{2cs}\right) = \frac{EA}{L}\left(\frac{Pc^2}{2c^3+1} - \frac{H}{2s}\right).$$

The component in x direction is bigger when $H \neq 0$ and $\alpha \to 0$ for $F^{(1)}$ and $F^{(3)}$

$$\lim_{\substack{\alpha \to 0\\ H \neq 0}} \pm\frac{H}{2s} = \pm\infty$$



$\varphi = 90$

$\cos 90 = 0$

$\sin 90 = 1$

$\frac{EA}{L}$

$$\bar{u}^{(2)} = T^{(2)} u^{(2)}$$

$$u^{(2)} = \left[u_{x_1} \; u_{y_1} \; u_{x_3} \; u_{y_3} \right]^T$$

$$= \left[\frac{H}{2cs^2} \; -\frac{P}{2c^3+1} \; 0 \; 0 \right]^T$$

$$\begin{bmatrix} \bar{u}_{x_1} \\ \bar{u}_{y_1} \\ \bar{u}_{x_3} \\ \bar{u}_{y_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{H}{2cs^2} \\ -\frac{P}{2c^3+1} \\ 0 \\ 0 \end{bmatrix}$$

$\bar{u}_{x_3} = 0$

$\bar{u}_{y_3} = 0$

$\bar{u}_{x_1} = -\frac{P}{2c^3+1}$

$\bar{u}_{y_1} = -\frac{H}{2cs^2}$

$$d^{(2)} = \bar{u}_{x_3} - \bar{u}_{x_1} = 0 - \left(-\frac{P}{2c^3+1} \right) = \frac{P}{2c^3+1}$$

$$F^{(2)} = \frac{EA}{L} \left(\frac{P}{2c^3+1} \right)$$

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