Assigment 3.1

1.- Given: $\lambda(E,\nu)$ to get: $E(\lambda,\mu)$

$\mu(E,\nu)$ $\nu(\lambda,\mu)$.

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad ①$$

$$\mu = G = \frac{E}{2(1+\nu)} \quad ② \rightarrow E = \mu\, 2(1+\nu) = 2\mu\left(1+\frac{\lambda}{2(\mu+\lambda)}\right)$$

$$= \cancel{2}\mu\left(\frac{2(\mu+\lambda)+\lambda}{\cancel{2}(\mu+\lambda)}\right)$$

$$E = \frac{\mu(2\mu+3\lambda)}{\mu+\lambda}$$

from ①: ② in ①

$$\lambda = \frac{2\mu\cancel{(1+\nu)}}{\cancel{(1+\nu)}(1-2\nu)}$$

$$\frac{(1-2\nu)}{\nu} = \frac{2\mu}{\lambda}$$

$$\frac{1}{\nu} - 2 = \frac{2\mu}{\lambda}$$

$$\frac{1}{\nu} = \frac{2\mu}{\lambda} + 2$$

$$\nu = \frac{1}{\frac{2\mu}{\lambda}+2} = \frac{1}{\frac{2\mu+2\lambda}{\lambda}} = \frac{\lambda}{2(\mu+\lambda)}$$

$$\nu = \frac{\lambda}{2(\mu+\lambda)}$$

2.

Plane stress:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ e_{xy}\end{bmatrix}.$$

$$= \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)\left(1-\left(\frac{\lambda}{2(\mu+\lambda)}\right)^2\right)}\begin{bmatrix}1 & \frac{\lambda}{2(\mu+\lambda)} & 0\\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0\\ 0 & 0 & \frac{1-\frac{\lambda}{2(\mu+\lambda)}}{2}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ e_{xy}\end{bmatrix}.$$

$$= \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)-\cancel{(\mu+\lambda)}\frac{\lambda^2}{4(\mu+\lambda)^{\cancel{2}}}}\begin{bmatrix}1 & \frac{\lambda}{2(\mu+\lambda)} & 0\\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0\\ 0 & 0 & \frac{1}{2}-\frac{\lambda}{4(\mu+\lambda)}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ e_{xy}\end{bmatrix}$$

$$= \frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}}\begin{bmatrix}1 & \frac{\lambda}{2(\mu+\lambda)} & 0\\ \frac{\lambda}{2(\mu+\lambda)} & 1 & 0\\ 0 & 0 & \frac{1}{2}-\frac{\lambda}{4(\mu+\lambda)}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ e_{xy}\end{bmatrix}$$

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Plain Strain.

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix}1 & \frac{\nu}{1-\nu} & 0\\ \frac{\nu}{1-\nu} & 1 & 0\\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ 2e_{xy}\end{bmatrix}$$

$$= \frac{\mu(2\mu+3\lambda)\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}{(\mu+\lambda)\left(1+\frac{\lambda}{2(\mu+\lambda)}\right)\left(1-2\left(\frac{\lambda}{2(\mu+\lambda)}\right)\right)}\begin{bmatrix}1 & \frac{\lambda}{2\mu+\lambda} & 0\\ \frac{\lambda}{2(\mu+\lambda)\cdot\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)} & 1 & 0\\ 0 & 0 & \frac{1-\frac{\lambda}{\mu+\lambda}}{2-\frac{\lambda}{\mu+\lambda}}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ 2e_{xy}\end{bmatrix}$$

$$\frac{\lambda}{2(\mu+\lambda)-\frac{\cancel{2(\mu+\lambda)}\lambda}{\cancel{2(\mu+\lambda)}}} = \frac{\lambda}{2\mu+2\lambda-\lambda} = \frac{\lambda}{2\mu+\lambda}$$

$$\frac{1-2\nu}{2(1-\nu)} = \frac{1-\cancel{2}\frac{\lambda}{\cancel{2}(\mu+\lambda)}}{2\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)} = \frac{1-\frac{\lambda}{\mu+\lambda}}{2-\frac{\cancel{2}\lambda}{\cancel{2}(\mu+\lambda)}} = \frac{1-\frac{\lambda}{\mu+\lambda}}{2-\frac{\lambda}{\mu+\lambda}}$$

$$\frac{\mu(2\mu+3\lambda)\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}{\left((\mu+\lambda)+\frac{\cancel{(\mu+\lambda)}\lambda}{2\cancel{(\lambda+\mu)}}\right)\left(1-\cancel{2}\left(\frac{\lambda}{\cancel{2}(\lambda+\mu)}\right)\right)} = \frac{\mu(2\mu+3\lambda)\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}{\left(\mu+\frac{3}{2}\lambda\right)\left(1-\left(\frac{\lambda}{\lambda+\mu}\right)\right)}$$

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \frac{\mu(2\mu+3\lambda)\left(1-\frac{\lambda}{2(\mu+\lambda)}\right)}{\left(\mu+\frac{3}{2}\lambda\right)\left(1-\frac{\lambda}{\lambda+\mu}\right)}\begin{bmatrix}1 & \frac{\lambda}{2\mu+\lambda} & 0\\ \frac{\lambda}{2\mu+\lambda} & 1 & 0\\ 0 & 0 & \frac{\lambda}{2\mu+\lambda}\end{bmatrix}\begin{bmatrix}e_{xx}\\ e_{yy}\\ e_{xy}\end{bmatrix}$$

3.

$$E = E_\lambda + E_\mu$$

$$E = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}$$ Strain-Stress Matrix.

$$E_{11} = E_{22} = \frac{E}{1-\nu^2} = \frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)\left(1-\left(\frac{\lambda}{2(\mu+\lambda)}\right)^2\right)} =$$

$$= \frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}}$$

$$E_{33} = \frac{1}{2}\frac{E}{1+\nu} = \frac{1}{2}\frac{\mu(2\mu+3\lambda)}{(\mu+\lambda)\left(1+\frac{\lambda}{2(\mu+\lambda)}\right)} = \frac{1}{2}\frac{\mu(2\mu+3\lambda)}{\mu+\frac{3}{2}\lambda}$$

$$E_{12} = \nu E_{11} = \left(\frac{\lambda}{2(\mu+\lambda)}\right)\left(\frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}}\right)$$

$$E_{13} = E_{23} = 0$$

$$E = \begin{bmatrix} \frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}} & \left(\frac{\lambda}{2(\mu+\lambda)}\right)\left(\frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}}\right) & 0 \\ \left(\frac{\lambda}{2(\mu+\lambda)}\right)\left(\frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}}\right) & \frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}} & 0 \\ 0 & 0 & \frac{\mu(2\mu+3\lambda)}{\mu+\lambda-\frac{\lambda^2}{4(\mu+\lambda)}} \end{bmatrix}$$

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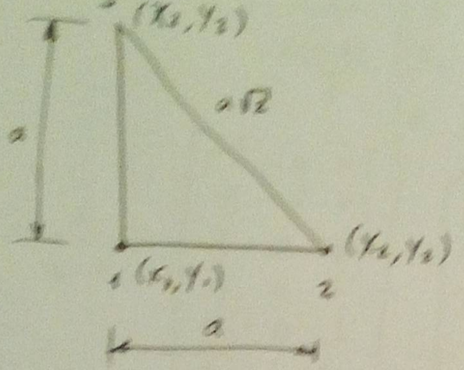
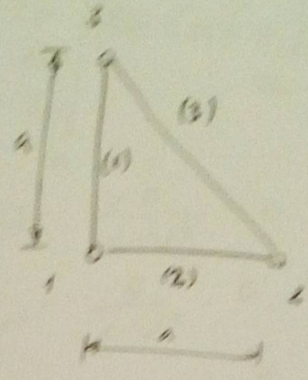
4.

$$E = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{1}{2}\frac{E}{1+\nu} \end{bmatrix}$$

Assignment 3.2



a)



b)

$A_1 = A_2 = A_3$

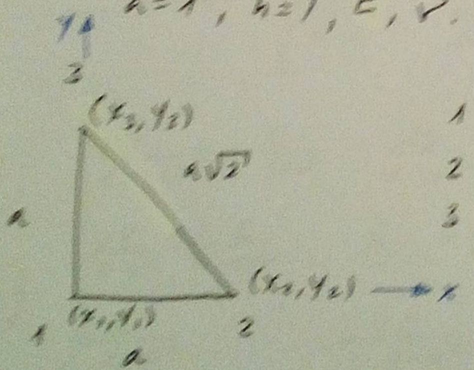
$\sqrt{a^2+a^2} = \sqrt{2a^2} = a\sqrt{2}$

1. $K_{3x} = ?$

$K_{bar} = ?$

for a) A plane linear Turner Triangle with the same dimensions.

$a = 1$, $h = 1$, E, ν. initially $\nu = 0$. $K_{tri} = ?$ Stiffness matrix.



	x	y
1	(0	, 0)
2	(a	, 0)
3	(0	, a)

$$2A = \det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$= (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3) + (x_1 y_2 - x_2 y_1)$$

$$= (a \cdot a - 0) + (0 \cdot 0 - 0 \cdot a) + (0 \cdot 0 - a \cdot 0)$$

$$= a^2 = 1^2 = 1$$

$x_{jk} = x_j - x_k$

$y_{jk} = y_j - y_k$

$$K = AhB^TEB = \frac{h}{\underbrace{4A}_{2(\underbrace{2A}_{1})}} \underbrace{\begin{bmatrix} y_{23} & 0 & x_{32} \\ 0 & x_{32} & y_{23} \\ y_{31} & 0 & x_{13} \\ 0 & x_{13} & y_{31} \\ y_{12} & 0 & x_{21} \\ 0 & x_{21} & y_{12} \end{bmatrix}}_{6\times3} \underbrace{\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{21} & E_{32} & E_{33} \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}}_{3\times6}$$

$$= \frac{1}{2\cdot1} \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{1}{2}\frac{E}{1+\nu} \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$y_{23} = 0 - a = -a = -1$

$x_{32} = 0 - a = -a = -1$

$y_{31} = a - 0 = a = 1$

$x_{13} = 0 - 0 = 0$

$y_{12} = 0 - 0 = 0$

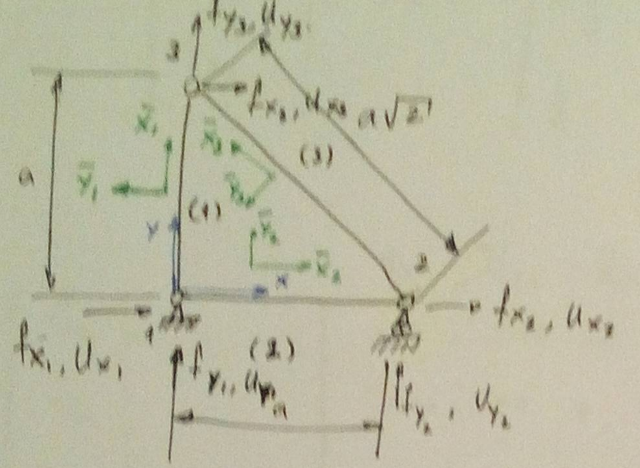
$x_{21} = a - 0 = a = 1$

$$= \frac{1}{2} \begin{bmatrix} -\frac{E}{1-\nu^2} & -\frac{\nu E}{1-\nu^2} & -\frac{1}{2}\frac{E}{1+\nu} \\ -\frac{\nu E}{1-\nu^2} & -\frac{E}{1-\nu^2} & -\frac{1}{2}\frac{E}{1+\nu} \\ \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{1}{2}\frac{E}{1+\nu} \\ 0 & 0 & \frac{1}{2}\frac{E}{1+\nu} \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \underbrace{\begin{bmatrix} \frac{E}{1-\nu^2}+\frac{1}{2}\frac{E}{1+\nu} & \frac{\nu E}{1-\nu^2}+\frac{1}{2}\frac{E}{1+\nu} & -\frac{E}{1-\nu^2} & -\frac{1}{2}\frac{E}{1+\nu} & -\frac{1}{2}\frac{E}{1+\nu} & -\frac{\nu E}{1-\nu^2} \\ \frac{\nu E}{1-\nu^2}+\frac{1}{2}\frac{E}{1+\nu} & \frac{E}{1-\nu^2}+\frac{1}{2}\frac{E}{1+\nu} & -\frac{\nu E}{1-\nu^2} & -\frac{1}{2}\frac{E}{1+\nu} & -\frac{1}{2}\frac{E}{1+\nu} & -\frac{E}{1-\nu^2} \\ -\frac{E}{1-\nu^2} & -\frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 & 0 & \frac{\nu E}{1-\nu^2} \\ -\frac{1}{2}\frac{E}{1+\nu} & -\frac{1}{2}\frac{E}{1+\nu} & 0 & \frac{1}{2}\frac{E}{1+\nu} & \frac{1}{2}\frac{E}{1+\nu} & 0 \\ -\frac{1}{2}\frac{E}{1+\nu} & -\frac{1}{2}\frac{E}{1+\nu} & 0 & \frac{1}{2}\frac{E}{1+\nu} & \frac{1}{2}\frac{E}{1+\nu} & 0 \\ -\frac{\nu E}{1-\nu^2} & -\frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 & 0 & \frac{E}{1-\nu^2} \end{bmatrix}}_{K_{tri}}$$

for b). A set of three bar elements placed over the edges of the triangular domain. The cross section for the bars are

$A_1 = A_2 = A_3 = A$

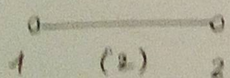


1.

$\varphi = 90°$ $\cos 90 = 0$ $\sin 90 = 1$

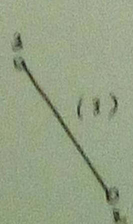
$$\frac{EA}{L} = \frac{EA}{a} = \frac{EA}{1} = EA$$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_3}^{(1)} \\ f_{y_3}^{(1)} \end{bmatrix} = EA \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_3} \\ u_{y_3} \end{bmatrix}$$



$\varphi = 0$ $\cos 0 = 1$ $\sin 0 = 0$ $\frac{EA}{L} = EA.$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \end{bmatrix} = EA \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \end{bmatrix}$$



$\varphi = 135°$ $\cos 135 = -\frac{\sqrt{2}}{2}$ $\sin 135 = \frac{\sqrt{2}}{2}$ $\frac{EA}{L} = \frac{EA}{a\sqrt{2}} = \frac{\sqrt{2}}{2}EA$

$c^2 = \frac{1}{2}$

$s^2 = \frac{1}{2}$ $sc = -\frac{1}{2}$

$$\begin{bmatrix} f_{x_2}^{(3)} \\ f_{y_2}^{(3)} \\ f_{x_3}^{(3)} \\ f_{y_3}^{(3)} \end{bmatrix} = \frac{\sqrt{2}}{2} EA \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \end{bmatrix}$$

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ f_{x_2} \\ f_{y_2} \\ f_{x_3} \\ f_{y_3} \end{bmatrix} = EA \underbrace{\begin{array}{c} \begin{array}{cccccc} u_{x_1} & u_{y_1} & u_{x_2} & u_{y_2} & u_{x_3} & u_{y_3} \end{array} \\ \begin{bmatrix} 0+1 & 0+0 & -1 & 0 & 0 & 0 \\ 0+0 & 1+0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1+\frac{\sqrt{2}}{4} & 0-\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \\ 0 & 0 & 0-\frac{\sqrt{2}}{4} & 0+\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ 0 & 0 & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} \\ 0 & 0 & \frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & -\frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} \end{bmatrix} \end{array}}_{K_{bar}} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \end{bmatrix}.$$

2.

initially $\nu = 0$.

K_{tri}:

$$\frac{1}{2} E \begin{bmatrix} \frac{3}{2} & \frac{3}{2} & -1 & -\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{3}{2} & 0 & -\frac{1}{2} & -\frac{1}{2} & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$K_{bar} \neq K_{tri}$

If. $A_2 = \frac{3}{2}$ and $A_3 = \sqrt{2}$ could be $\underline{K_{bar}}$ more similar to $\underline{K_{tri}}$

3. This two stiffnes matrices are not equal because the strain in K_{bar} at the node 1 are $U_{X_1} = U_{Y_1} = 0$. In K_{tri} the 3 nodes have strain different to zero.

4.

If $\nu \neq 0$:

$$1 - \nu^2 \neq 0 \quad \text{and} \quad 1 + \nu \neq 0$$

$$\nu \neq -1$$

$$1 = \nu^2$$

$$\nu = \pm\sqrt{1}$$

$$\nu < \pm 1$$

$\therefore \quad -1 < \nu < 1 \longrightarrow$ The value of ν is limited.