COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 1

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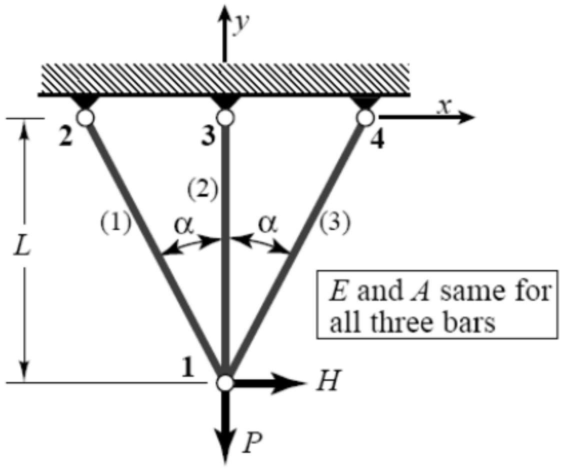
<u>Assignment 1</u>

On "The Direct Stiffness Method":

Consider the truss problem defined in the Figure. All geometric and material properties: L, α, E and A, as well as the applied forces P and H, are to be kept as variables. This truss has 8 degrees of freedom, with six of them removable by the fixed-displacement conditions at nodes 2, 3 and 4. This structure is statically indeterminate as long as $\alpha \neq 0$.



(a) Show that the master stiffness equations are

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symm} & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{x1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

in which $c = \cos\alpha$ and $s = \sin\alpha$. Explain from physics why the 5th row and column contain only zeros.

We have that for a typical system of coordinates, the stiffness matrix for a truss element with 4 DOF for any angle α is

$$\boldsymbol{K}^e = \frac{E^e A^e}{L^e}\begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix}$$

with the convention mentioned before. Given that element (1) is at a $\pi/2+\alpha$ angle, element (2) at a $\pi/2$ angle and element (3) at a $\pi/2-\alpha$ angle, the local stiffness matrices are:

$$\boldsymbol{K}^{(1)} = \frac{cEA}{L}\begin{bmatrix} s^2 & -cs & -s^2 & cs \\ -cs & c^2 & cs & -c^2 \\ -s^2 & cs & s^2 & -cs \\ cs & -c^2 & -cs & c^2 \end{bmatrix}$$

$$\boldsymbol{K}^{(2)} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{K}^{(3)} = \frac{cEA}{L}\begin{bmatrix} s^2 & cs & -s^2 & -cs \\ cs & c^2 & -cs & -c^2 \\ -s^2 & -cs & s^2 & cs \\ -cs & -c^2 & cs & c^2 \end{bmatrix}$$

considering the identities: $\sin(\pi/2+\alpha) = c$, $\cos(\pi/2+\alpha) = -s$, $\sin(\pi/2-\alpha) = c$ and $\cos(\pi/2-\alpha) = s$; and that by trigonometry, $L^{(1)} = L^{(3)} = L/c$. To form the global matrix we need to expand the local matrices to the size of the global matrix, keeping the components in the places that correspond to each displacement. Using the numeration proposed in the problem, the matrices are:

$$\boldsymbol{K}^{(1)} = \frac{EA}{L}\begin{bmatrix} c\,s^2 & -c^2\,s & -c\,s^2 & c^2\,s & 0 & 0 & 0 & 0 \\ -c^2\,s & c^3 & c^2\,s & -c^3 & 0 & 0 & 0 & 0 \\ -c\,s^2 & c^2\,s & c\,s^2 & -c^2\,s & 0 & 0 & 0 & 0 \\ c^2\,s & -c^3 & -c^2\,s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{K}^{(2)} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{K}^{(3)} = \frac{EA}{L}\begin{bmatrix} c\,s^2 & c^2\,s & 0 & 0 & 0 & 0 & -c\,s^2 & -c^2\,s \\ c^2\,s & c^3 & 0 & 0 & 0 & 0 & -c^2\,s & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c\,s^2 & -c^2\,s & 0 & 0 & 0 & 0 & c\,s^2 & c^2\,s \\ -c^2\,s & -c^3 & 0 & 0 & 0 & 0 & c^2\,s & c^3 \end{bmatrix}$$

adding the three matrices we finally have

$$\boldsymbol{K} = \frac{EA}{L}\begin{bmatrix} 2\,c\,s^2 & 0 & -c\,s^2 & c^2\,s & 0 & 0 & -c\,s^2 & -c^2\,s \\ 0 & 2\,c^3+1 & c^2\,s & -c^3 & 0 & -1 & -c^2\,s & -c^3 \\ -c\,s^2 & c^2\,s & c\,s^2 & -c^2\,s & 0 & 0 & 0 & 0 \\ c^2\,s & -c^3 & -c^2\,s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -c\,s^2 & -c^2\,s & 0 & 0 & 0 & 0 & c\,s^2 & c^2\,s \\ -c^2\,s & -c^3 & 0 & 0 & 0 & 0 & c^2\,s & c^3 \end{bmatrix}$$

which is the matrix that we are proposed. The null 5th row and column is due to the nature of the truss element. Truss elements only work in tension and compression, so it can't take bending. Since element (2) is pinned at the top and is a vertical element, the only deformation allowed to the bar is in the local axis of the bar, this means, on the vertical global axis, which makes zero the contribution of the horizontal displacement of the 3rd node. Every other displacement has a contribution due to the fact that the bars are inclined.

(b) Apply the BCs and show the 2-equation modified stiffness system.

Since nodes 2, 3 and 4 are pinned, the movement is restricted in both directions, yielding displacement zero. This means that only u_{x1} and u_{y1} are to be computed, so reducing the system:

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

This is achieved by removing the rows corresponding to the known displacements. The columns are also removed since now they are being multiplied by zero.

(c) Solve for the displacements u_{x1} and u_{y1}. Check that the solution makes physical sense for the limit cases $\alpha \to 0$ and $\alpha \to \pi/2$. Why does u_{x1} "blow up" if $H \neq 0$ and $\alpha \to 0$?

The solution is

$$\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \frac{L}{AE}\begin{bmatrix} 1/2cs^2 & 0 \\ 0 & 1/(1+2c^3) \end{bmatrix}\begin{bmatrix} H \\ -P \end{bmatrix} = \begin{bmatrix} HL/(2AEcs^2) \\ -PL/(AE(1+2c^3)) \end{bmatrix}$$

When $\alpha \to 0$:

$$\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} HL/0 \\ -PL/(3AE) \end{bmatrix}$$

which makes physical sense, because we are considering as if the three bars were pointing down, thus having one bar with three times the area of one, which is expressed on the value of u_{y1}. In this case u_{x1} blows up because the element is in an unstable equilibrium due to being pinned instead of fixed, which makes it a mechanism (less unknowns that equations). This makes more sense if we imagine the bar pointing upwards instead of hanging: any force $H \neq 0$ would make the bar to fall down due to the fact that the support does not resist the moment created by the force, and this is expressed in the equations as an indeterminate solution.

When $\alpha \to \pi/2$:

$$\begin{bmatrix} u_{x1} \\ u_{y1} \end{bmatrix} = \begin{bmatrix} 0/0 \\ 0 \end{bmatrix}$$

This case makes no physical sense, as we would have two bars parallel to the ceiling, so $L = 0$, which again leads to something indeterminate on the horizontal direction and zero on the vertical one.

(d) Recover the axial forces in the three members. Partial answer: $F^{(3)} = -H/(2s) + Pc^2/(1+2c^3)$. Why do $F^{(1)}$ and $F^{(3)}$ "blow up" if $H \neq 0$ and $\alpha \to 0$?

Since we have the global displacements, we can obtain the forces of the elements on their local axes given the rotation matrix

$$\boldsymbol{T} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

$$\boldsymbol{T}^{(1)} = \begin{bmatrix} -s & c & 0 & 0 \\ -c & -s & 0 & 0 \\ 0 & 0 & -s & c \\ 0 & 0 & -c & -s \end{bmatrix} \quad \boldsymbol{T}^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad \boldsymbol{T}^{(3)} = \begin{bmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{bmatrix}$$

Using this, we obtain the local displacements:

$$\bar{\boldsymbol{u}}^{(e)} = \boldsymbol{T}^{(e)}\boldsymbol{u}^{(e)}$$

$$\bar{\boldsymbol{u}}^{(1)} = \begin{bmatrix} -cPL/(AE(1+2c^3)) - HL/(2AEcs) \\ sPL/(AE(1+2c^3)) - HL/(2AEs^2) \\ 0 \\ 0 \end{bmatrix} \quad \bar{\boldsymbol{u}}^{(2)} = \begin{bmatrix} -PL/(AE(1+2c^3)) \\ -HL/(2AEcs^2) \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{\boldsymbol{u}}^{(3)} = \begin{bmatrix} -cPL/(AE(1+2c^3)) + HL/(2AEcs) \\ -sPL/(AE(1+2c^3)) - HL/(2AEs^2) \\ 0 \\ 0 \end{bmatrix}$$

Finally, we get the axial force with the Force-Deformation relation

$$F = \frac{AE}{L}\delta$$

where $\delta = u_f - u_i$.

$$\begin{aligned} \delta_1 &= \bar{u}_{2x} - \bar{u}_{1x} = cPL/(AE(1+2c^3)) + HL/(2AEcs) \\ \delta_2 &= \bar{u}_{3x} - \bar{u}_{1x} = PL/(AE(1+2c^3)) \\ \delta_3 &= \bar{u}_{4x} - \bar{u}_{1x} = cPL/(AE(1+2c^3)) - HL/(2AEcs) \end{aligned}$$

$$\begin{aligned} F^{(1)} &= cP/(1+2c^3) + H/(2cs) \\ F^{(2)} &= P/(1+2c^3) \\ F^{(3)} &= cP/(1+2c^3) - H/(2cs) \end{aligned}$$

Again, the reason why $F^{(1)}$ and $F^{(3)}$ blow up when $\alpha \to 0$ is by the fact that the bars are all on the vertical axis, so we have effectively a bar with area $3A$, and this is reflected on the fact that, when $H = 0$, every bar is in tension with a third of the total force. When $H \neq 0$, we have an unstable system due to the pin on top, so we get an indeterminate solution for every element dependent on H.

Assignment 2

Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node 5 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 4-5. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

Since we still have truss elements that are only affected by the angle, local matrices are the same for elements (1) and (2), while for elements (3) and (4) we get similar matrices to the former element (3) matrix, with the exception of the length of the element. Thus, the stiffness matrix for elements (3) and (4) are

$$\boldsymbol{K}^{(3)} = \boldsymbol{K}^{(4)} = \frac{2cEA}{L}\begin{bmatrix} s^2 & cs & -s^2 & -cs \\ cs & c^2 & -cs & -c^2 \\ -s^2 & -cs & s^2 & cs \\ -cs & -c^2 & cs & c^2 \end{bmatrix}$$

The expanded forms are

$$\boldsymbol{K}^{(1)} = \begin{bmatrix} \frac{AEcs^2}{L} & -\frac{AEc^2s}{L} & -\frac{AEcs^2}{L} & \frac{AEc^2s}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{AEc^2s}{L} & \frac{AEc^3}{L} & \frac{AEc^2s}{L} & -\frac{AEc^3}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{AEcs^2}{L} & \frac{AEc^2s}{L} & \frac{AEcs^2}{L} & -\frac{AEc^2s}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{AEc^2s}{L} & -\frac{AEc^3}{L} & -\frac{AEc^2s}{L} & \frac{AEc^3}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{K}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{AE}{L} & 0 & 0 & 0 & -\frac{AE}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{AE}{L} & 0 & 0 & 0 & \frac{AE}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{K}^{(3)} = \begin{bmatrix} \frac{2AEcs^2}{L} & \frac{2AEc^2s}{L} & 0 & 0 & 0 & 0 & -\frac{2AEcs^2}{L} & -\frac{2AEc^2s}{L} & 0 & 0 \\ \frac{2AEc^2s}{L} & \frac{2AEc^3}{L} & 0 & 0 & 0 & 0 & -\frac{2AEc^2s}{L} & -\frac{2AEc^3}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{2AEcs^2}{L} & -\frac{2AEc^2s}{L} & 0 & 0 & 0 & 0 & \frac{2AEcs^2}{L} & \frac{2AEc^2s}{L} & 0 & 0 \\ -\frac{2AEc^2s}{L} & -\frac{2AEc^3}{L} & 0 & 0 & 0 & 0 & \frac{2AEc^2s}{L} & \frac{2AEc^3}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\boldsymbol{K}^{(4)} = \begin{bmatrix}
\frac{2\,A\,E\,c\,s^2}{L} & \frac{2\,A\,E\,c^2\,s}{L} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2\,A\,E\,c\,s^2}{L} & -\frac{2\,A\,E\,c^2\,s}{L} \\
\frac{2\,A\,E\,c^2\,s}{L} & \frac{2\,A\,E\,c^3}{L} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c^3}{L} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{2\,A\,E\,c\,s^2}{L} & -\frac{2\,A\,E\,c^2\,s}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\,A\,E\,c\,s^2}{L} & \frac{2\,A\,E\,c^2\,s}{L} \\
-\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c^3}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\,A\,E\,c^2\,s}{L} & \frac{2\,A\,E\,c^3}{L}
\end{bmatrix}
$$

So the global stiffness matrix is

$$
\boldsymbol{K} = \begin{bmatrix}
\frac{5\,A\,E\,c\,s^2}{L} & \frac{3\,A\,E\,c^2\,s}{L} & -\frac{A\,E\,c\,s^2}{L} & \frac{A\,E\,c^2\,s}{L} & 0 & 0 & -\frac{2\,A\,E\,c\,s^2}{L} & -\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c\,s^2}{L} & -\frac{2\,A\,E\,c^2\,s}{L} \\
\frac{3\,A\,E\,c^2\,s}{L} & \frac{A\,E}{L} + \frac{5\,A\,E\,c^3}{L} & \frac{A\,E\,c^2\,s}{L} & -\frac{A\,E\,c^3}{L} & 0 & -\frac{A\,E}{L} & -\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c^3}{L} & -\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c^3}{L} \\
-\frac{A\,E\,c\,s^2}{L} & \frac{A\,E\,c^2\,s}{L} & \frac{A\,E\,c\,s^2}{L} & -\frac{A\,E\,c^2\,s}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{A\,E\,c^2\,s}{L} & -\frac{A\,E\,c^3}{L} & -\frac{A\,E\,c^2\,s}{L} & \frac{A\,E\,c^3}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{A\,E}{L} & 0 & 0 & 0 & \frac{A\,E}{L} & 0 & 0 & 0 & 0 \\
-\frac{2\,A\,E\,c\,s^2}{L} & -\frac{2\,A\,E\,c^2\,s}{L} & 0 & 0 & 0 & 0 & \frac{2\,A\,E\,c\,s^2}{L} & \frac{2\,A\,E\,c^2\,s}{L} & 0 & 0 \\
-\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c^3}{L} & 0 & 0 & 0 & 0 & \frac{2\,A\,E\,c^2\,s}{L} & \frac{2\,A\,E\,c^3}{L} & 0 & 0 \\
-\frac{2\,A\,E\,c\,s^2}{L} & -\frac{2\,A\,E\,c^2\,s}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\,A\,E\,c\,s^2}{L} & \frac{2\,A\,E\,c^2\,s}{L} \\
-\frac{2\,A\,E\,c^2\,s}{L} & -\frac{2\,A\,E\,c^3}{L} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2\,A\,E\,c^2\,s}{L} & \frac{2\,A\,E\,c^3}{L}
\end{bmatrix}
$$

Which again has the null column due to the phenomenon explained before. Since we only allow movement in node 1 and 4, we delete all columns and row except the 1st, 2nd, 7th and 8th column:

$$
\boldsymbol{K}^r = \frac{AE}{L} \begin{bmatrix}
5\,c\,s^2 & 3\,c^2\,s & -2\,c\,s^2 & -2\,c^2\,s \\
3\,c^2\,s & 1 + 5\,c^3 & -2\,c^2\,s & -2\,c^3 \\
-2\,c\,s^2 & -2\,c^2\,s & 2\,c\,s^2 & 2\,c^2\,s \\
-2\,c^2\,s & -2\,c^3 & 2\,c^2\,s & 2\,c^3
\end{bmatrix}
$$

The system associated with this stiffness matrix can't be solved due to the fact that this matrix is singular, so we have no unique solution to the problem. Physically, this is again explained by the fact that the structure is behaving as a mechanism, in which any perturbance, as small as it is, can set an unstable motion on the structure. This is due to the fact that the joint in the middle of the member is a redundant support, since the forces can be directly transmitted from one element to the other without it.