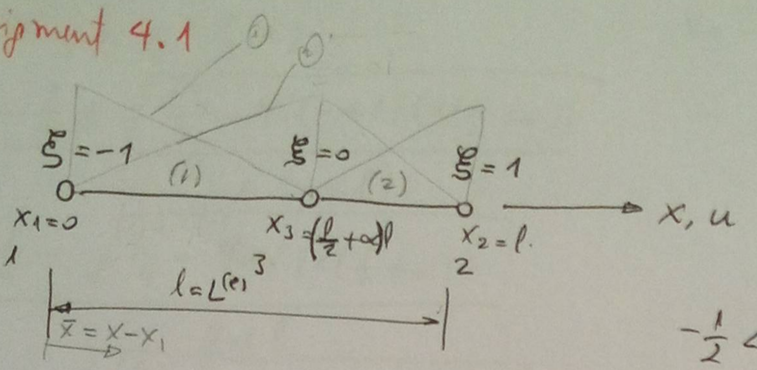
CSMD. Jorge Alvarez 06/03/17 1/1

Assigment 4.1



$l = x_1 - x_2$

$-\frac{1}{2} < \alpha < \frac{1}{2}$

$x_1=0 \quad x_2=l \quad x_3=\left(\frac{1}{2}+\alpha\right)l$

$$\begin{bmatrix} 1 \\ x \\ u \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} N_1^e \\ N_2^e \\ N_3^e \end{bmatrix}$$

$$x = \sum_{i=1}^{3} x_i N_i^e = [x_1 \;\; x_2 \;\; x_3] \begin{bmatrix} N_1^e \\ N_2^e \\ N_3^e \end{bmatrix} \qquad \frac{\partial N_i^e}{\partial x} = \frac{\partial N_i^e}{\partial \xi} \cdot \frac{\partial \xi}{\partial x}$$

$$u = \sum_{i=1}^{3} u_i N_i^e = [u_1 \;\; u_2 \;\; u_3] \begin{bmatrix} N_1^e \\ N_2^e \\ N_3^e \end{bmatrix}$$

Local (isoparametric) coordinates

$$N_1(\xi) = -\frac{\xi(1-\xi)}{2}$$

$$N_2(\xi) = \frac{\xi(1+\xi)}{2}$$

$$N_3(\xi) = 1-\xi^2$$

Isoparametric mapping.

$$x = \sum_{i=1}^{3} N_i(\xi)\, x_i$$

$$x = -\frac{\xi(1-\xi)}{2} \cdot x_1 + \frac{\xi(1+\xi)}{2} x_2 + (1-\xi^2) x_3$$

$$x = -\frac{\xi(1-\xi)}{2} \cdot 0 + \frac{\xi(1+\xi)}{2} \cdot l + (1-\xi^2)\left(\frac{1}{2}+\alpha\right) l$$

$$x = \frac{\xi \cdot l}{2} + \cancel{\frac{\xi^2}{2} l} + \frac{1}{2} l + \alpha l - \cancel{\frac{\xi^2}{2} l} - \xi^2 \alpha l$$

$$x(\xi) = \underbrace{l\left(\frac{1}{2}+\alpha\right)}_{c} + \underbrace{\frac{l}{2}}_{b}\xi \underbrace{-\alpha l}_{a}\xi^2$$

CSMD. Jorge Alvarez 08/03/17 1/.

Inverse Mapping. $\xi(x)$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\xi_{(x)} = \frac{-\frac{l}{2} \pm \sqrt{\frac{l^2}{4} - 4.(-\alpha l)(l(\frac{1}{2}+\alpha))}}{2.(-\alpha l)}$$

$$\xi(x) = \frac{\frac{l}{2} - \sqrt{\frac{l^2}{4} + 4\alpha l(l(\frac{1}{2}+\alpha))}}{2\alpha l}$$

$$\xi(x) = \frac{\frac{l}{2} - \sqrt{\frac{l^2}{4} + \cancel{4}^{2}\alpha\frac{l^2}{\cancel{2}} + 4\alpha^2 l^2}}{2\alpha l}$$

$$\xi(x) = \frac{\frac{l}{2} - l\sqrt{\frac{1}{4} + 2\alpha + 4\alpha^2}}{2\alpha l}$$

$$\xi(x) = \frac{\frac{1}{2} - \sqrt{\frac{1}{4} + 2\alpha + 4\alpha^2}}{2\alpha}$$

$$x = \sum_{i=1}^{3} N_i(\xi) x_i$$

$$\frac{dN_1(\xi)}{d\xi} = -\frac{1}{2}((1-\xi)+\xi(-1))$$
$$= -\frac{1}{2}(1-2\xi)$$
$$= \xi - \frac{1}{2}$$

$$\frac{dx}{d\xi} = \sum_{i=1}^{3} \frac{dN_i(\xi)}{d\xi} x_i = J \text{ (Jacobian of mapping).}$$

$$\frac{dN_2(\xi)}{d\xi} = \frac{1}{2}((1+\xi)+\xi)$$
$$= \frac{1}{2}(1+2\xi) = \xi + \frac{1}{2}$$

$$\frac{dN_3(\xi)}{d\xi} = -2\xi$$

$$J = \frac{dN_1(\xi)}{d\xi}x_1 + \frac{dN_2(\xi)}{d\xi}x_2 + \frac{dN_3(\xi)}{d\xi}x_3$$

$$J = (\xi - \tfrac{1}{2})0 + (\xi + \tfrac{1}{2})l + (-2\xi)(\tfrac{1}{2}+\alpha)l$$

$$J = \cancel{\xi l} + \frac{1}{2}l - \cancel{2\xi . \frac{1}{2}}. l - 2\xi\alpha l$$

$$J = \frac{1}{2}l - 2\xi\alpha l = l\left(\frac{1}{2} - 2\xi\alpha\right)$$

$$\frac{dx}{d\xi} = J = l\left(\frac{1}{2} - 2\xi\alpha\right)$$

a) $-\frac{1}{4} < \alpha < \frac{1}{4}$ for $-1 \leq \xi \leq 1$

$\alpha = -\frac{1}{4}$

$$J = l\left(\frac{1}{2} - 2\xi\left(-\frac{1}{4}\right)\right)$$

$$= l\underbrace{\left(\frac{1}{2} + \underbrace{\frac{\xi}{2}}_{<\frac{1}{2}}\right)}_{>0} \quad \xi \to -\frac{1}{3.999}$$

$\therefore J > 0$

$\alpha = \frac{1}{4}$

$$J = l\left(\frac{1}{2} - 2\xi\left(\frac{1}{4}\right)\right)$$

$$= l\underbrace{\left(\frac{1}{2} - \underbrace{\frac{\xi}{2}}_{<\frac{1}{2}}\right)}_{>0} \quad \xi \to \frac{1}{3.999}$$

$\therefore J > 0$

b) $\alpha = 0$

$$J = l\left(\frac{1}{2} - \underbrace{2\cdot\xi\cdot 0}_{0}\right)$$

$$J = \frac{1}{2}$$

2.-

$B = ?$

$$B = \left[\frac{dN_1}{dx} \quad \frac{dN_2}{dx} \quad \frac{dN_3}{dx}\right] \; ; \quad \frac{dN_i(\xi)}{dx} = \frac{dN_i(\xi)}{d\xi}\underbrace{\frac{d\xi}{dx}}_{\frac{1}{J}}$$

$$B = \frac{1}{J}\left[\frac{dN_1(\xi)}{d\xi} \quad \frac{dN_2(\xi)}{d\xi} \quad \frac{dN_3(\xi)}{d\xi}\right]$$

$$\frac{dN_i(\xi)}{dx} = \frac{1}{J}\frac{dN_i(\xi)}{d\xi}$$

$$B = \frac{1}{l\left(\frac{1}{2} - 2\xi\alpha\right)}\left[\xi - \frac{1}{2} \quad \xi + \frac{1}{2} \quad -2\xi\right]$$

CSMD

3.

Jorge Alvarez

08/03/17

3/

$$K^e = \int_0^1 E A B^T B \, dx = \int_{-1}^{1} E A B^T B \, J \, d\xi$$

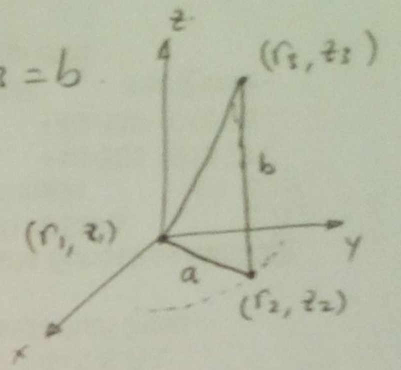
$$\frac{dN_i(\xi)}{dx} = \frac{1}{J} \frac{dN_i(\xi)}{d\xi} \longrightarrow dx = J \, d\xi$$

$$0 \leq x \leq 1 \qquad -1 \leq \xi \leq 1$$

CSMD. Jorge Alvarez 08/03/17 4/

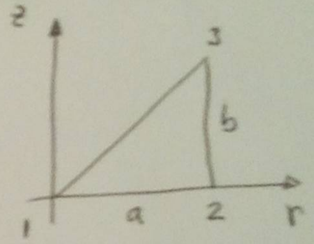
Assignment 4.2

$r_1 = 0\ ,\ r_2 = r_3 = a\ ,\ z_1 = z_2 = 0\ ,\ z_3 = b$



with $\nu = 0$:

$$\mathbf{E} = E \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$



Iso-P.

$$\begin{bmatrix} 1 \\ r \\ z \\ u_r \\ u_z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & a & a \\ 0 & 0 & b \\ u_{r1} & u_{r2} & u_{r3} \\ u_{z1} & u_{z2} & u_{z3} \end{bmatrix} \begin{bmatrix} N_1^{(e)} \\ N_2^{(e)} \\ N_3^{(e)} \end{bmatrix}$$

shape function matrix

$$\begin{bmatrix} u_r \\ u_z \end{bmatrix} = \begin{bmatrix} N_1^e & 0 & N_2^e & 0 & N_3^e & 0 \\ 0 & N_1^e & 0 & N_2^e & 0 & N_3^e \end{bmatrix} u^e = N^e u^e$$

$$u^e = \begin{bmatrix} u_{r1} & u_{z1} & u_{r2} & u_{z2} & u_{r3} & u_{z3} \end{bmatrix}^T$$

$$1 = N_1^e + N_2^e + N_3^e \longrightarrow N_1^e = 1 - \left(\frac{r}{a} - \frac{z}{b}\right) - \frac{z}{b} = 1 - \frac{r}{a}$$

$$r = aN_2^e + aN_3^e \longrightarrow N_2^e = \frac{r - a\frac{z}{b}}{a} = \frac{r}{a} - \frac{z}{b}$$

$$z = bN_3^e \longrightarrow N_3^e = \frac{z}{b}$$

$$\begin{bmatrix} 1 \\ r \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ r_1 & r_2 & r_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ r \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & a & a \\ 0 & 0 & b \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix}$$

$$\begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\end{bmatrix}=\frac{1}{2A}\begin{bmatrix} r_2z_3-r_3\cancel{z_2} & \cancel{z_2}-z_3 & r_3-r_2\\ r_3\cancel{z_1}-r_1z_3 & z_3-\cancel{z_1} & \cancel{r_1}-r_3\\ r_1z_2-r_2\cancel{z_1} & \cancel{z_1}-\cancel{z_2} & r_2-\cancel{r_1}\end{bmatrix}\begin{bmatrix}1\\ r\\ z\end{bmatrix}$$

$$\begin{bmatrix}\zeta_1\\ \zeta_2\\ \zeta_3\end{bmatrix}=\frac{1}{\cancel{2}\cdot\frac{a\cdot b}{\cancel{2}}}\begin{bmatrix} ab & -b & 0\\ 0 & b & -a\\ 0 & 0 & a\end{bmatrix}\begin{bmatrix}1\\ r\\ z\end{bmatrix} \qquad A=\frac{a\cdot b}{2}.$$

$$1-\frac{r}{a}+0=\zeta_1=N_1^e$$

$$\frac{r}{a}-\frac{z}{b}=\zeta_2=N_2^e$$

$$\frac{z}{b}=\zeta_3=N_3^e$$

$$B^e=\frac{1}{2A}\begin{bmatrix} z_{23} & 0 & z_{31} & 0 & z_{12} & 0\\ 0 & r_{32} & 0 & r_{13} & 0 & r_{21}\\ \frac{2A\zeta_1}{r} & 0 & \frac{2A\zeta_2}{r} & 0 & \frac{2A\zeta_3}{r} & 0\\ r_{32} & z_{23} & r_{13} & z_{31} & r_{21} & z_{12}\end{bmatrix}$$

$$B^e=\frac{1}{ab}\begin{bmatrix} -b & 0 & b & 0 & 0 & 0\\ 0 & 0 & 0 & -a & 0 & a\\ \frac{ab}{r}-b & 0 & b-\frac{az}{br} & 0 & \frac{az}{r} & 0\\ 0 & -b & -a & b & a & 0\end{bmatrix}$$

$$\frac{2A\zeta_1}{r}=\frac{a\cdot b\cdot\left(1-\frac{r}{a}\right)}{r}=\frac{ab-b\cdot r}{r}=\frac{ab}{r}-b$$

$$\frac{2A\zeta_2}{r}=\frac{ab\left(\frac{r}{a}-\frac{z}{b}\right)}{r}=\frac{br-\frac{az}{b}}{r}=b-\frac{az}{rb}$$

$$\frac{2A\zeta_3}{r}=\frac{ab\left(\frac{z}{b}\right)}{r}=\frac{az}{r}$$

CJMD

Jorge Alvarez

08/03/17. 6/6

$$K^e = \frac{1}{4A^2} \int_{\Omega^e} r \begin{bmatrix} z_{23} & 0 & z_{31} & 0 & z_{12} & 0 \\ 0 & r_{32} & 0 & r_{13} & 0 & r_{21} \\ \frac{2A\zeta_1}{r} & 0 & \frac{2A\zeta_2}{r} & 0 & \frac{2A\zeta_3}{r} & 0 \\ r_{32} & z_{23} & r_{13} & z_{31} & r_{21} & z_{12} \end{bmatrix}^T \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} \\ E_{12} & E_{22} & E_{23} & E_{24} \\ E_{13} & E_{23} & E_{33} & 0 \\ E_{14} & E_{24} & 0 & E_{44} \end{bmatrix}$$

$$\begin{bmatrix} z_{23} & 0 & z_{31} & 0 & z_{12} & 0 \\ 0 & r_{32} & 0 & r_{13} & 0 & r_{21} \\ \frac{2A\zeta_1}{r} & 0 & \frac{2A\zeta_2}{r} & 0 & \frac{2A\zeta_3}{r} & 0 \\ r_{32} & z_{23} & r_{13} & z_{31} & r_{21} & E_{12} \end{bmatrix} d\Omega.$$

$$= \frac{1}{\cancel{4} \cdot \frac{a^2b^2}{\cancel{4}}} \int_{\Omega^e} r \begin{bmatrix} -b & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & a \\ \frac{ab}{r} - b & 0 & b - \frac{az}{br} & 0 & \frac{az}{r} & 0 \\ 0 & -b & -a & b & a & 0 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} -b & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & a \\ \frac{ab}{r} - b & 0 & b - \frac{az}{br} & 0 & \frac{az}{r} & 0 \\ 0 & -b & -a & b & a & 0 \end{bmatrix} d\Omega$$

$$= \frac{1}{a^2b^2} \int_{\Omega^e} r \underbrace{\begin{bmatrix} -b & 0 & \frac{ab}{r} - b & 0 \\ 0 & 0 & 0 & -b \\ b & 0 & b - \frac{az}{br} & -a \\ 0 & -a & 0 & b \\ 0 & 0 & \frac{az}{r} & a \\ 0 & a & 0 & 0 \end{bmatrix}}_{6\times4} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}}_{4\times4}.$$

$$\underbrace{\begin{bmatrix} -b & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & a \\ \frac{ab}{r} - b & 0 & b - \frac{az}{br} & 0 & \frac{az}{r} & 0 \\ 0 & -b & -a & b & a & 0 \end{bmatrix}}_{4\times6} d\Omega.$$

CSMD. Jorge Alvarez 10/03/17 1/

$$= \frac{1}{a^2b^2}\int_{\Omega^e} r \begin{bmatrix} -b & 0 & \frac{ab}{r}-b & 0 \\ 0 & 0 & 0 & -\frac{b}{2} \\ b & 0 & b-\frac{az}{br} & -\frac{a}{2} \\ 0 & -a & 0 & \frac{b}{2} \\ 0 & 0 & \frac{az}{r} & \frac{a}{2} \\ 0 & a & 0 & 0 \end{bmatrix}_{6\times 4} \begin{bmatrix} -b & 0 & b & 0 & 0 & 0 \\ 0 & 0 & 0 & -a & 0 & a \\ \frac{ab}{r}-b & 0 & b-\frac{az}{br} & 0 & \frac{az}{r} & 0 \\ 0 & -b & -a & b & a & 0 \end{bmatrix}_{4\times 6} d\Omega$$

$$= \frac{1}{a^2b^2}\int_{\Omega^e} r \begin{bmatrix} b^2+\left(\frac{ab}{r}-b\right)^2 & 0 & -b+\left(\frac{ab}{r}-b\right)\left(b-\frac{az}{br}\right) & 0 & \left(\frac{ab}{r}-b\right)\frac{az}{r} & 0 \\ 0 & \frac{b^2}{2} & +\frac{ab}{2} & -\frac{b^2}{2} & -\frac{ab}{2} & 0 \\ -b^2+\left(b-\frac{az}{br}\right)\left(\frac{ab}{r}-b\right) & \frac{ab}{2} & b^2+\left(b-\frac{az}{br}\right)^2-\frac{a^2}{2} & -\frac{ab}{2} & \left(b-\frac{az}{br}\right)\left(\frac{az}{r}\right) & 0 \\ 0 & -\frac{b^2}{2} & -\frac{ab}{2} & a^2+\frac{b^2}{2} & \frac{ab}{2} & -a^2 \\ \left(\frac{az}{r}\right)\left(\frac{ab}{r}-b\right) & -\frac{ab}{2} & \left(\frac{az}{r}\right)\left(b-\frac{az}{br}\right)-\frac{a^2}{2} & \frac{ab}{2} & \left(\frac{az}{r}\right)^2+\frac{a^2}{2} & 0 \\ 0 & 0 & 0 & -a^2 & 0 & a^2 \end{bmatrix} d\Omega$$

row:

2 : $\frac{b^2}{2}+\frac{ab}{2}-\frac{b^2}{2}-\frac{ab}{2}=0$

4 : $-\frac{b^2}{2}-\frac{ab}{2}+a^2+\frac{b^2}{2}+\frac{ab}{2}-a^2=0$

6 : $-a^2+a^2=0.$

column:

2 : $\frac{b^2}{2}+\frac{ab}{2}-\frac{b^2}{2}-\frac{ab}{2}=0$

4 : $-\frac{b^2}{2}-\frac{ab}{2}+a^2+\frac{b^2}{2}+\frac{ab}{2}-a^2=0$

6 : $-a^2+a^2=0.$

3. Jorge Alvarez 10/03/17

$$f^{(e)} = ?$$

$$b = [0, -g]^T = [b_r(r,z), b_z(r,z)]$$ One dimensional body force field.

$$f^{(e)} = \int_{\Omega^e} N^T b \, r \, d\Omega \longrightarrow \text{node force vector}$$

$$= \int_{\Omega^e} \begin{bmatrix} N_1^e & 0 \\ 0 & N_1^e \\ N_2^e & 0 \\ 0 & N_2^e \\ N_3^e & 0 \\ 0 & N_3^e \end{bmatrix} \begin{bmatrix} b_r \\ b_z \end{bmatrix} r \, d\Omega = \int_{\Omega^e} \begin{bmatrix} N_1^e b_r \\ N_1^e b_z \\ N_2^e b_r \\ N_2^e b_z \\ N_3^e b_r \\ N_3^e b_z \end{bmatrix} r \, d\Omega$$

$$f^{(e)} = \int_{\Omega^e} \begin{bmatrix} 0 \\ (1 - \frac{r}{a})(-g) \\ 0 \\ (\frac{r}{a} - \frac{z}{b})(-g) \\ 0 \\ -\frac{z}{b} \cdot g \end{bmatrix} r \, d\Omega.$$