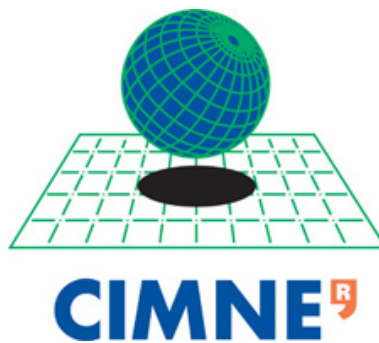

FINAL INTERNSHIP REPORT

PROPER GENERALIZED DECOMPOSITION USING JULIA



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1 General Introduction

Tensor decomposition or separation is regarded as a way of representing a multidimensional tensor as a linear combination of rank-1 tensors. These rank-1 tensors are represented by tensor product of a collection of vectors. Therefore, the data in a multidimensional tensor can be collected in a group of 1d vectors. Several methods are available and being developed for this process. Proper Generalized Decomposition (PGD) is a new promising method that is used for the process.

PGD is a computational method to solve high-dimensional boundary value problems. Its philosophy is to provide a separated representation of the multidimensional solution. This is done using a greedy approach along with alternated direction scheme using Least squared approximation. Such approach can then be used for tensor decomposition.

2 Description of the internship

The internship topic was building a routine for tensor separation and compression as well as other tensor operations using Julia language, which is a high-level programming language specially designed for numerical analysis. The internship involved learning the Julia language through tutorials and the documentation. I had to understand the differences between Julia and MATLAB in order to adapt to coding in Julia. Understanding and learning the algorithm for tensor separation using Proper Generalized Decomposition (PGD) was also part of the performed tasks. This involves reading the related journal papers critically and understanding the logic and thinking of the algorithm. This phase took from 3-4 weeks.

In the next phase, I was focusing on coding the routine in Julia. That took long time and several trials from writing and running the code then tracing if there are any errors. Then, I tried

to make it more efficient by finding other shorter ways to reach the same target. At the end, the routines were then built for tensor separation and compression and other operations. Moreover, the routines were tested with some examples and compared with the already existing MATLAB routine. This phase took the rest of the internship duration.

3 Working Environment

The working environment was very constructive. The working place was very neat and enjoyable to work in. We got to work in a large room in the library. It was so quiet and enjoyable to work along with the other interns. This environment helped me clear my mind and focus on what I am doing. There were many places and small restaurants near the library so that we can have a lunch break in.

My advisor was so helpful and supportive. We used to contact through email and we had some meetings in which he cleared all my doubts with thoughtful answers. Moreover, he was supportive and usually gives me constructive feedback. This environment made me enthusiastic about the work and research more in the field.

4 Obstacles

There were several obstacles in the internship both technical and logistical. The technical ones were mainly adapting to the new language and understanding the main differences from MATLAB in the logic itself. One of these differences is that the arrays are not copied when assigned to another variable. Another difference is that the language doesn't automatically create an array in an assignment statement which is different from MATLAB. There were several issues in the beginning of coding because of these differences. Then, I managed to find where I have problems and fixed them.

The logistical issues that I faced was mainly to organize the work time along with the course tasks. Some weeks, I had many assignments to do along with the internship work, therefore, I had to stay long for other days to compensate the time lost doing the courses tasks. However, I managed to do well in both the classes and the internship.

5 Sample of the done work (Tensor Separation)

5.1 Input

The input is a multidimensional tensor F of order d . The size of the tensor in each dimension is given by n_i , where $i = 1, 2, \dots, d$, and n_i can be different in each dimension.

5.2 Output

The output is a separable format as a summation of M terms. Each term is composed of the tensorial product of d unit vectors of dimension n_i , where $i = 1, 2, \dots, d$. Each term is scaled by

a value σ^m , where $m=1,2,\dots,M$.

$$F = \sum_{m=1}^M \sigma^m f_1^m \otimes f_2^m \otimes \dots \otimes f_d^m$$

This is stored in Julia as two variables. The first is a vector of size M having the values of σ_m . The second is a cell matrix of size $d \times m$ storing the unit vectors.

5.3 Algorithm

The algorithm is the same given in "Algebraic PGD for tensor separation and compression: an algorithmic approach".

5.4 Examples and comparison with MATLAB code

5.4.1 1st Example

The first example to be tested is a 2d (7×3) matrix given as:

$$F = \begin{bmatrix} 3 & 2 & 1 \\ 80 & 15 & -2 \\ 4 & 8 & -8 \\ 15 & 8 & -80 \\ 2 & -5 & 90 \\ 9 & 15 & -80 \\ 4 & 8 & -70 \end{bmatrix}$$

The Julia subroutine provided the following results

$$\sigma = [162.765 \quad 81.9121 \quad 9.42642 \quad 2.6734 \times 10^{-15}]$$

The MATLAB code provided the following results

$$\sigma = [162.765 \quad 81.912079 \quad 9.42642 \quad 1.57 \times 10^{-15}]$$

The last mode is different in the values of σ and the sectional data, but it's almost zero and the difference is in the MATLAB and Julia accuracy.

5.4.2 2nd Example

The second example is 8d matrix ($4 \times 5 \times 2 \times 6 \times 8 \times 5 \times 3 \times 2$) full of ones. The Julia routine provided the following results.

$$\sigma = [240 \quad 3.158 \times 10^{-14}]$$

The MATLAB provides the following results.

$$\sigma = [240 \quad 8.52 \times 10^{-14}]$$

Same sectional results were obtained in both cases.

6 Reflection and Conclusion

The internship was a very positive experience for me. I learned how to organize my schedule in order not to miss anything. I learned to work under stress when I had several tasks to do. These skills can help me in the future in research or in my future job.

In conclusion, the internship developed my skills by learning a new coding language: Julia and by learning about PGD. The environment was so motivating to learn and it was easy to communicate with the professor for questions and recommendations.

The internship introduced me to a new promising technique that is more efficient than classical methods such as Finite Elements. However, PGD is still a new technique that needs further development. That is why I am motivated to devote my thesis project to try solving one of the problems that is still faced by PGD.