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Internship report

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The aim of this report is to provide a detailed account of the objective and tasks conducted during my internship period. The internship was titled as follows: Didactic modeling of beams. This internship was carried under the supervision of Dr. Riccardo Rossi at CIMNE.

The main objective of this internship was to conduct an analysis on beams in a manner that illustrates their behavior in order to create guide for educative purposes. Such analysis was conducted using the finite elements method using three dimensional elements. The analysis was conducted on beams of different cross-sectional shape that are subjected to different loading condition, each to illustrate a certain behavior. The analysis was structured as follows:

Cross-sectional shape	Loading type
Rectangular	Pure bending
I-shape	Pure bending
L-shape	Self-weight on the centroid
L-shape	Self-weight on the shear center
Hollow rectangle	Torsion
Hollow rectangle with a slit	Torsion

In order to construct the shape of the cross-section engineering standards for beams were consulted. Afterwards, a CAD model was constructed, and the analysis was carried out using GID as a pre and post processor.

For the pure bending cases, a linearly distributed load was applied on the cross-section of the beam at its right most fixing point; taking a zero value at the center, a positive value at the top surface and a negative value on the bottom surface. A similar load was applied on the left most fixing point; however, the loads signs were flipped. This had the effect of mimicking a moment couple which is required for a pure bending case.

For the self-weight cases, the point of application of the such load differed. Due to the cross-sectional shape having only one axis of symmetry, the point of application would result in different loading condition for the beam. When applied at the centroid, shear effects start to appear due to torsion in addition to the standard bending effects. Thus, by applying the self-weight on the horizontal portion of the L-shape only (the shear center) the shear effects are canceled. Both ends of the beam where held fixed.

For the torsion cases, in order to mimic such effect, loads having a direction in the global positive x-axis and loads having a direction in the global negative x-axis where applied on the top and bottom surfaces of the cross-section respectively; while, loads having a direction in the global positive y-axis and loads having a direction in the global negative y-axis where applied on the right and left surfaces of the cross-section respectively. Such load couples would have a torsional effect on the beam. The opposite side of the beam was held fixed.

The results and discussion of the above-mentioned cases are described in the attached document.

In order to better illustrate the results obtained from the finite elements analysis, interactive 3D files of the results were to be exported. Such files would be included in a PDF file that could be viewed by the students on their laptops without requiring any additional software. Due to the inability of GID to directly export to export directly to a format recognized by Acrobat reader several conversions had to be made. Such conversions where made possible using the Mesh Lab software. Where a VRML model would be exported from GID in order to conserve the required color gradient of the results on the model; then, Mesh Lab would be used to convert this file to the u3d format recognized by the PDF editor. The final results are shown in the attached PDF file.

Didactic modeling of beams

Introduction

This document aims to provide an overview of the behavior of beams under some loading conditions. The first case is that of beam with a square cross-section. This beam is initially subjected to a compressive load and then it is subjected to a moment that results in pure bending. The second case is that of an I beam subjected to pure bending. The cross-sectional area and the applied moment in this case are almost identical to the ones applied in the previous case. The effect of the changing cross-sectional shape is thus observed on the induced stresses in the beam. The third case to be analyzed is that of an L beam subjected to self-weight. In an initial analysis, the self-weight is applied at the center of gravity of the beam; while in a second analysis, self-weight is applied in a different location. The effect of the change in the point of application of self-weight is observed on the displacement of the beam. The third and final case is that of hollow beam with a square cross-section subjected to torsion. Two geometrical configurations are to be implemented: a normal hollow beam with a square cross-section and another with the same configuration but with a slit running through the length of one of its faces. The effect of such geometrical modification is to be observed on the behavior of the beam.

In this analysis, an infinitesimal strain setting is assumed where the beams are not subjected to large deformations. The loads are also applied in such a manner to retain the analysis in the elastic region. All dimensions used are in millimeters. Steel is the material of choice for this study where its material properties are shown in the Appendix.

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Square cross-section beam under compression and pure bending

In this section, a beam with a square cross-section is to be analyzed. First, it is subjected to an axial loading, namely compression. Then, a pure bending loading scenario is to be conducted. The dimensions of the beam and its cross-section are shown in Figure 1.

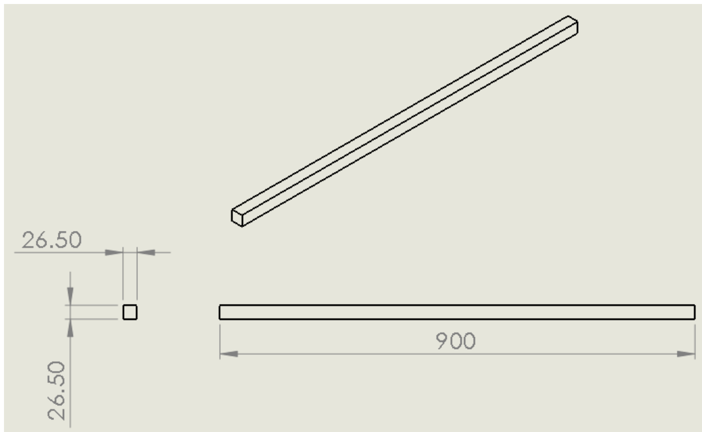

Figure 1: Dimensions of the square cross-section beam (mm)

Axial loading

A compressive axial load is applied to one side of the beam while the opposite side is held fixed. The normal stresses, σ, induced in the beam under such loading conditions could be computed according to the following equation:

$$\sigma = \frac{P}{A}$$

Where: σ is the normal stress, P is the applied axial load and A is the cross-sectional area of the beam.

The normal stress distribution across the beam is shown in Figure 2.

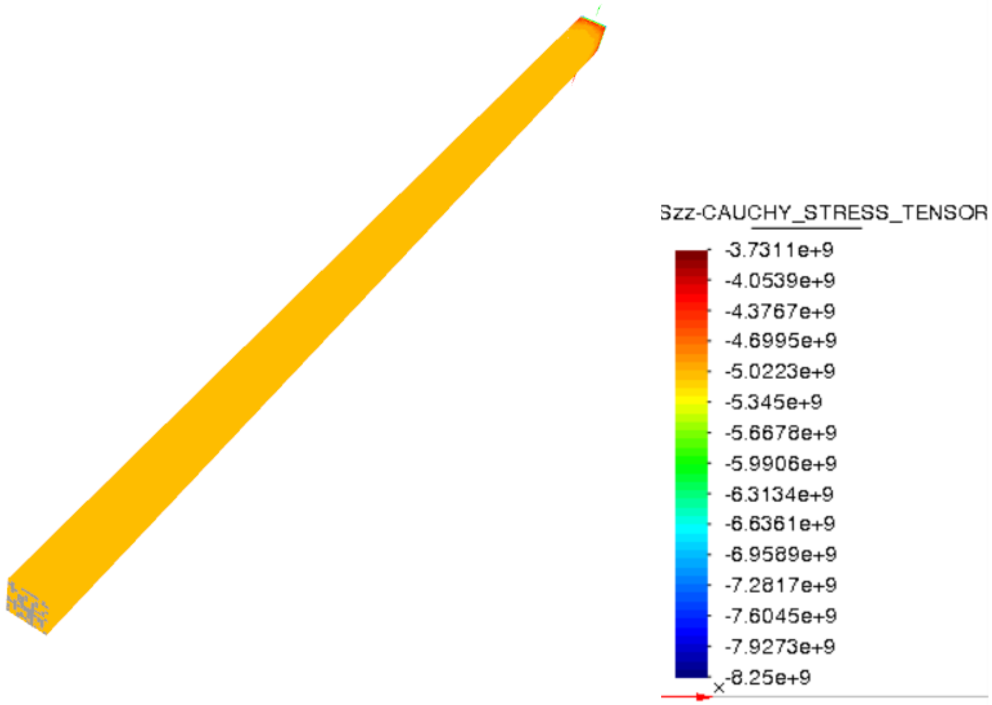

Figure 2: normal stress distribution of the square beam under compression

In addition, due to the compressive load, the beam undergoes axial displacement in the direction of the load. The first step to computing the displacement is to compute the strain. According to Hook's law, strain (ε) is proportional to the stress as follows:

$$\sigma = E\,\varepsilon$$

Where: ε is the axial strain and E is Young's modulus.

By using the Young's modulus corresponding to the used material, the strain could be computed. Moreover, displacement could be computed as follows:

$$\varepsilon = \frac{d}{l}$$

Where: d is the axial displacement and l is the initial longitudinal length of the beam.

The displacement of the beam under the applied compressive load is shown in Figure 3.

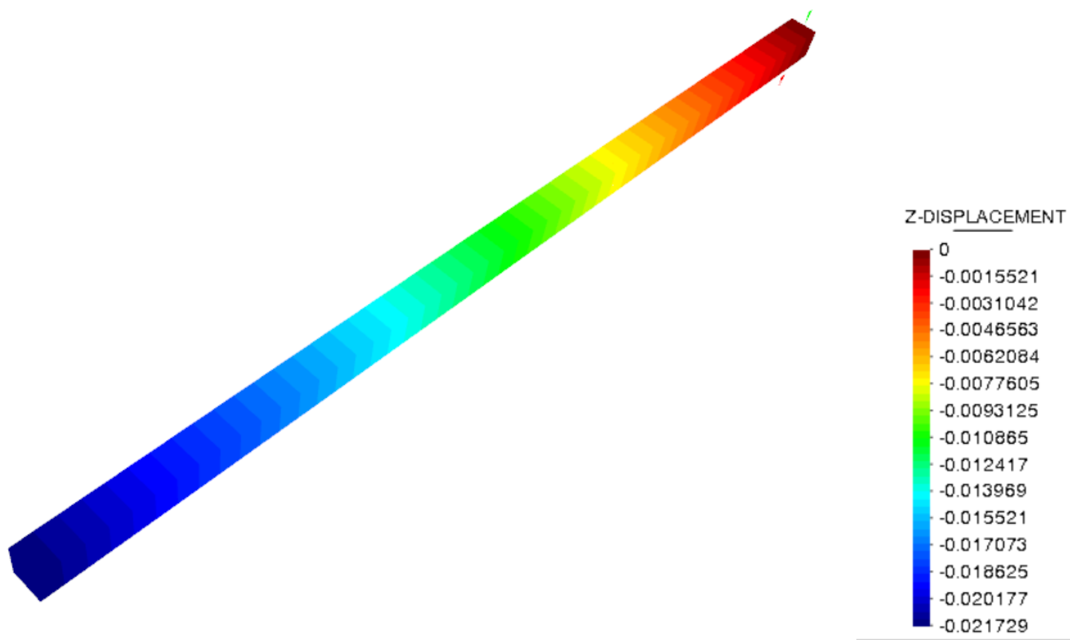

Figure 3: longitudinal displacement of square beam under compression

Pure bending

Now the case of pure bending is being considered. In order to achieve such loading scenario, opposite couples (bending moment) are applied to the beam from opposite sides along the longitudinal direction. It could be noted that the internal forces in any cross-section of the beam under pure bending are equivalent to a bending moment. This entails the fact that the beam will bend uniformly, thus any longitudinal line before applying an external bending moment will have a constant curvature afterwards.

In the case of pure bending, there exists a surface where the stress σ and the strain ε are zero. Such a surface is called the neutral surface and in the case of the symmetric square cross-section used in this example it coincides with horizontal plane of symmetry. The normal stress in the beam under pure bending could be calculated using the following formula:

$$\sigma = \frac{My}{I}$$

Where: M is the applied bending moment, y is the vertical distance from the neutral surface, and I is moment of inertia (second moment) corresponding to the beam's cross-section.

The normal stress distribution is shown in Figure 4.

Figure 4: Normal stress distribution of the square beam under pure bending σ_{zz}

It could be observed that the stresses are zero along the middle plane while it increases with opposite signs further from the center. The maximum value is reached at the top and bottom surfaces where the top surface is subjected to tension and the bottom surface to compression. It could be noted that the normal stress distribution is constant for all cross-sections of the beam. This due to the fact that the bending moment is constant along the beam in the case of pure bending. Moreover, shear is the derivative of the bending moment thus constant bending moment results in zero shear as shown in Figure 5.

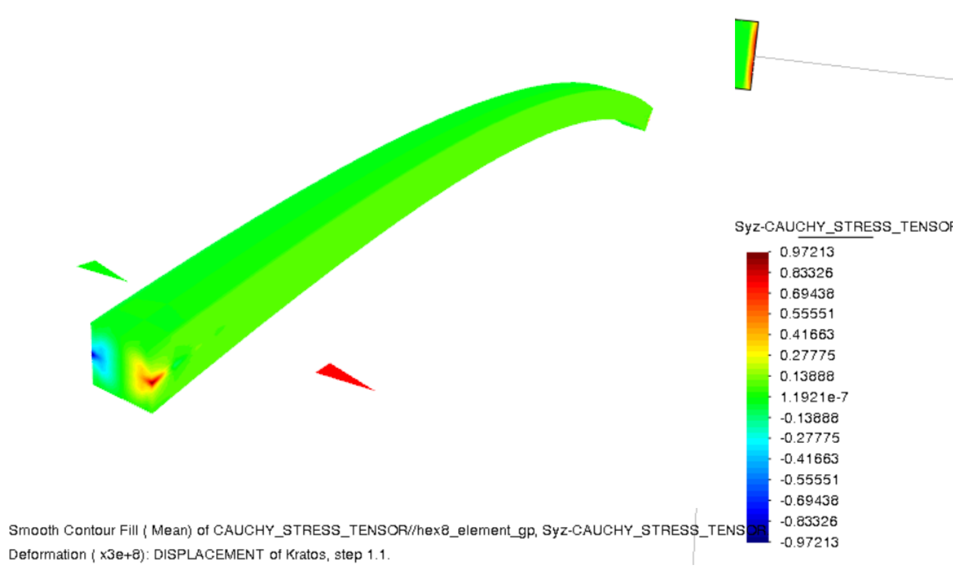

Figure 5: Shear stress distribution of the square beam under pure bending τ_{xy}

I cross-section beam under pure bending

In this section, a beam with a I cross-section is considered. The dimensions of the cross-section follow the IPE 80 standards. The dimensions are shown in Figure 6 where the fillets where omitted for simplicity. It should be noted that the cross-sectional area of the this I beam is almost equal to that of the square beam used in the previous section.

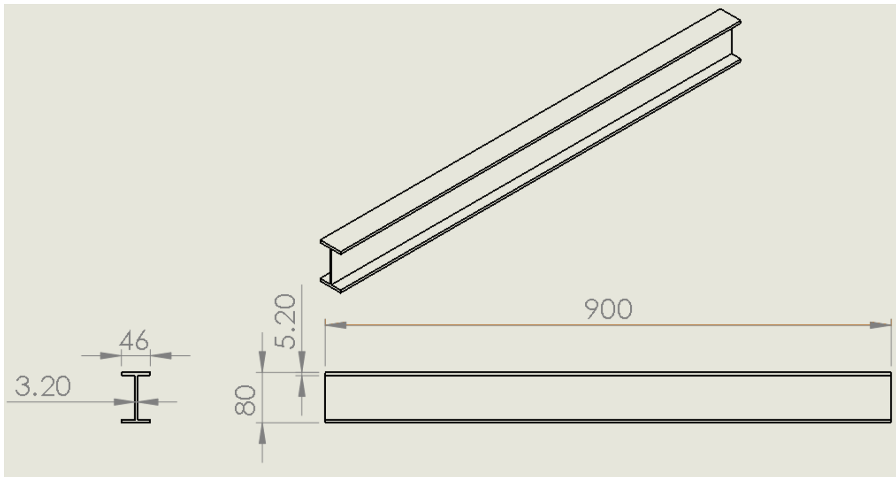

Figure 6: Dimensions of the I beam cross-section (mm)

The beam is subject to a bending moment resulting in a pure bending scenario. The normal stress distribution is shown in Figure 7.

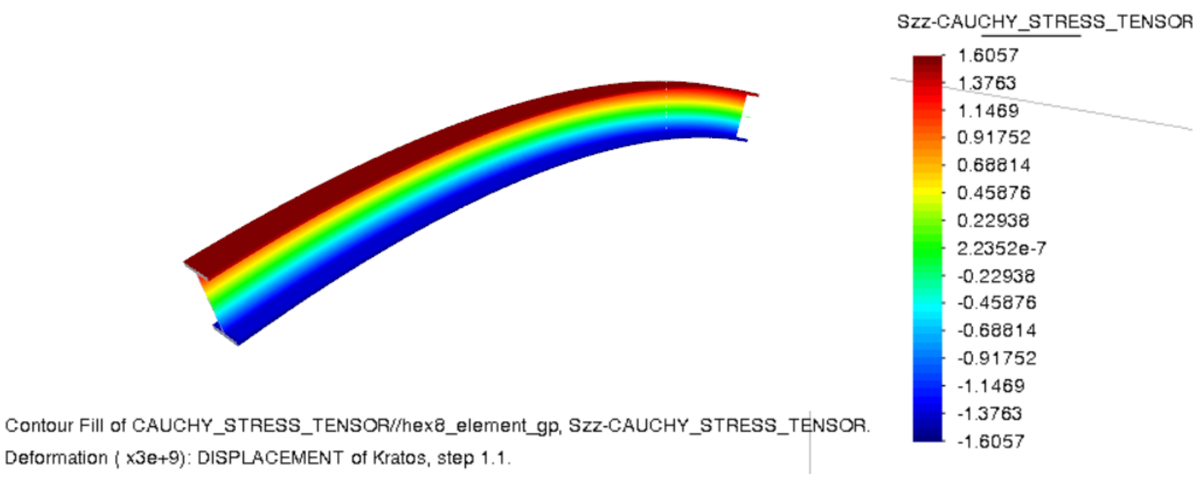

Figure 7: Normal stress distribution of the I beam under pure bending σ_{zz}

It could be observed that, similar to the square cross-section beam, the stress σ and the strain ε are zero at the neutral surface which coincides with the horizontal axis of symmetry of the beam. In addition, the maximum tensile and compressive stresses occur at the top and bottom surfaces respectively.

However, the main difference between these two cross-sectional shapes is that, for the same applied moment and cross-sectional area, the stresses induced in the I beam are smaller compared to the ones induced in the square beam. This is due to the fact that the I beam possesses a larger moment inertia. Again, similar to the square cross-section, the shear stress is zero across the beam as shown in Figure 8.

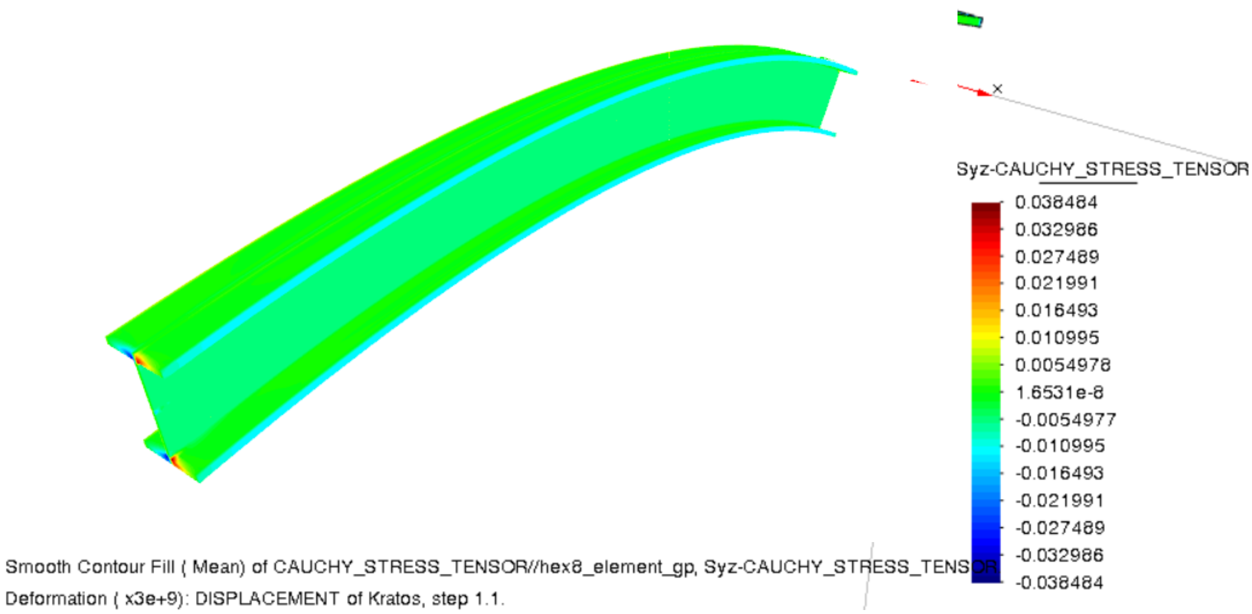

Figure 8: Shear stress distribution of the I beam under pure bending τ_{xy}

L cross-section beam

In this section, a beam with an L cross-section is to be analyzed. The beam is subjected to self-weight; however, the location of its application varies. The dimensions of the cross-section follow the DIN 1025 standards. The dimensions are shown in Figure 9 where the fillets where omitted for simplicity.

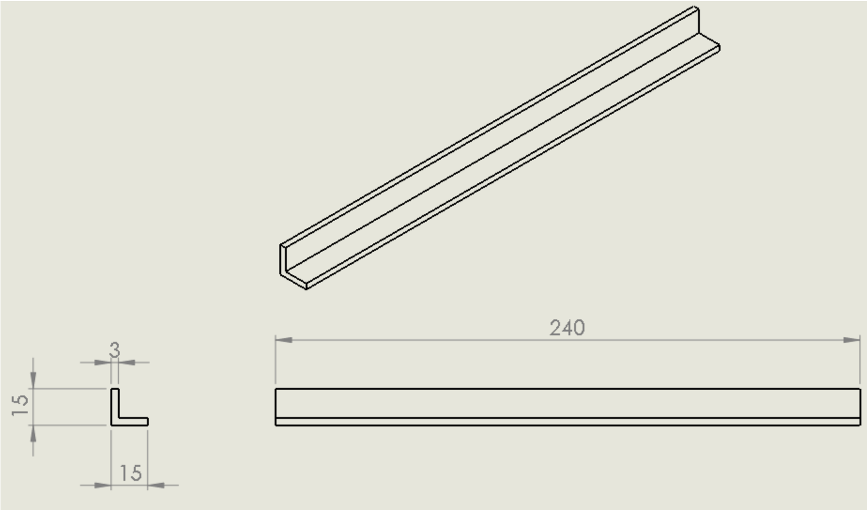

Figure 9: Dimensions of the L beam cross-section (mm)

The first case to be studied is where the weight of the beam is applied at its center of gravity (CG). When positioned in this orientation, the L beam does not possess a vertical plane of symmetry. In addition, the CG of the beam lies outside of its geometry. This situation results in the beam being subjected to an eccentric load. Thus, the beam will both bend and twist as a result to this loading condition. The effect of combining both bending and twisting could be seen in the displacement of the beam in the x, y and z direction as shown in Figure 10, Figure 11 and Figure 12 respectively.

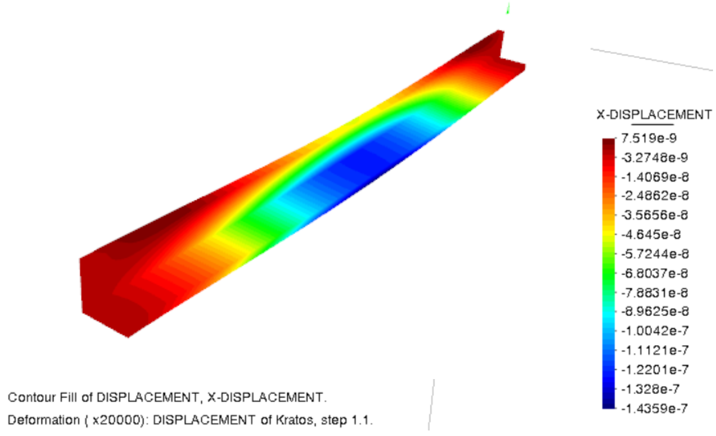

Figure 10: x-displacement of L beam under eccentric self-weight

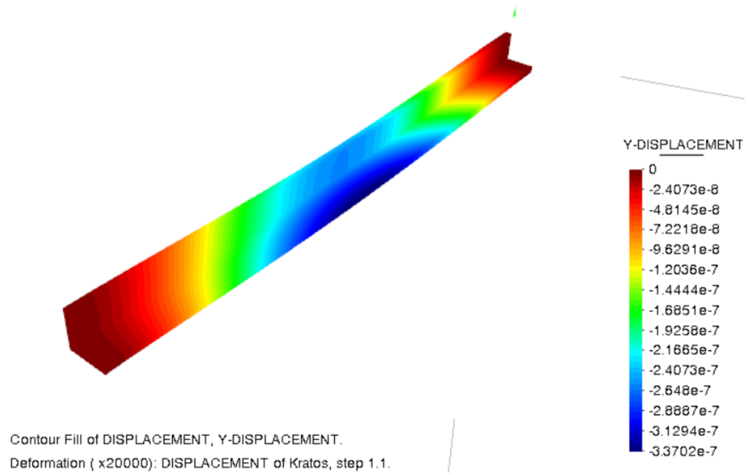

Figure 11: y-displacement of L beam under eccentric self-weight

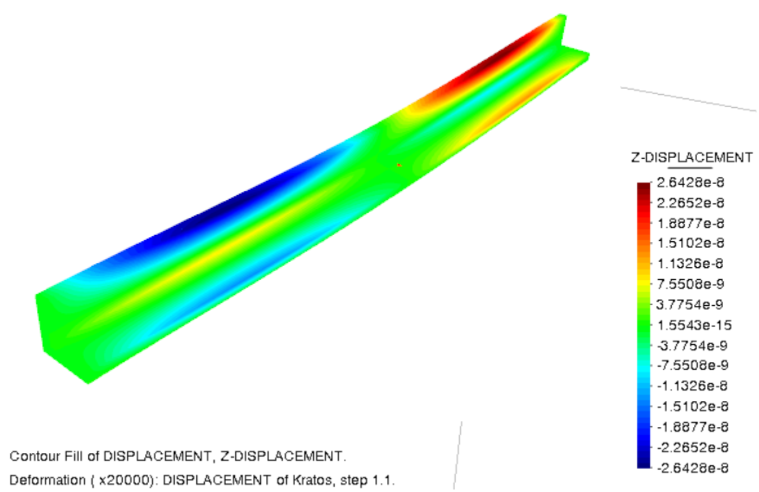

Figure 12: z-displacement of L beam under eccentric self-weight

The second case to be studied is where the weight of the beam is not applied at the CG. More specifically it would be applied at a point that would eliminate the effect of twisting. This would result in the beam being subjected to a bending moment only. Such a point is called the shear center of the cross-section. In the case of the L cross-section, the shear center lies in the corner created by the horizontal and vertical slabs. The effect of applying the self-weight on the shear center could be seen in the displacement of the beam in the x, y and z direction as shown in Figure 13, Figure 14 and Figure 15 respectively.

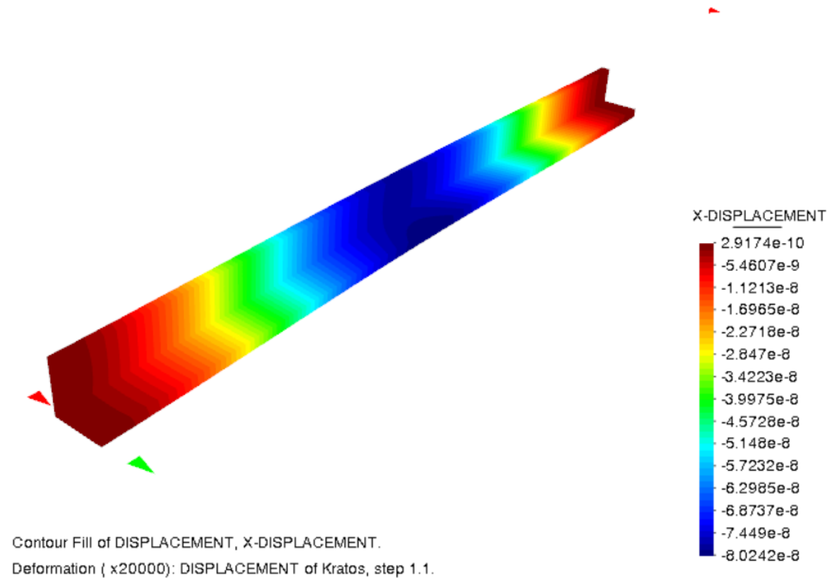

Figure 13: x-displacement of L beam under self-weight applied at the shear center

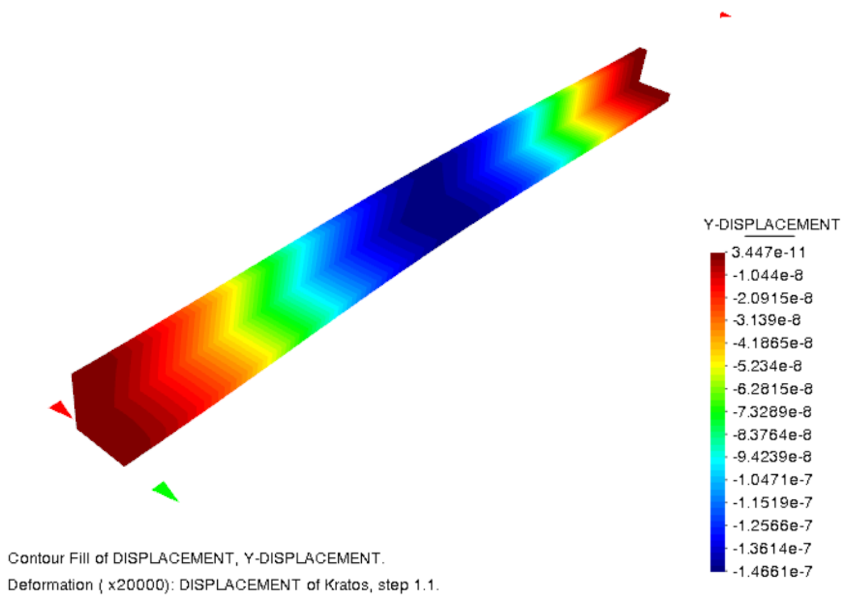

Figure 14: y-displacement of L beam under self-weight applied at the shear center

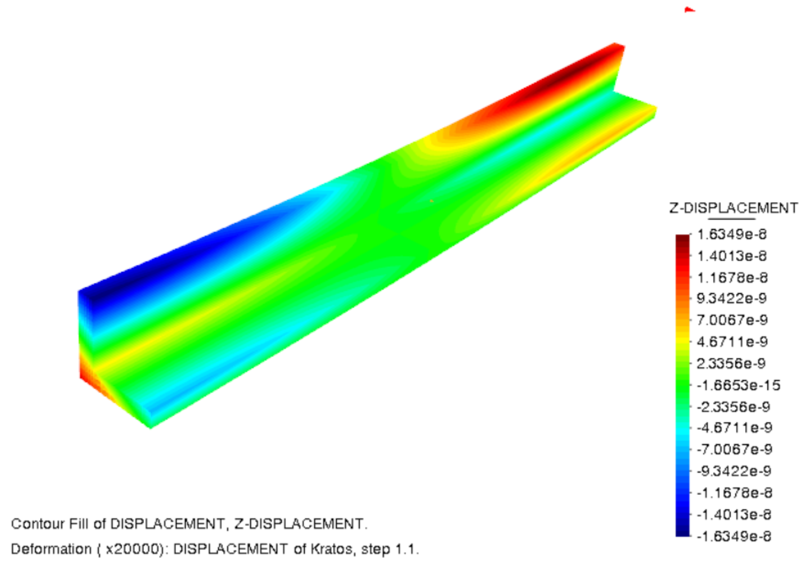

Figure 15: z-displacement of L beam under self-weight applied at the shear center

Hollow square cross-section beam under torsion

In this section, a beam with a rectangular cross-section is considered. The dimensions of the cross-section follow the EN 10219 standards; however, two geometrical configurations are to be implemented: a beam following the dimensional standards and another using the same dimensions but with a slit running across the length of one of the sides. The dimensions of both configurations are shown in Figure 16 and Figure 17 respectively.

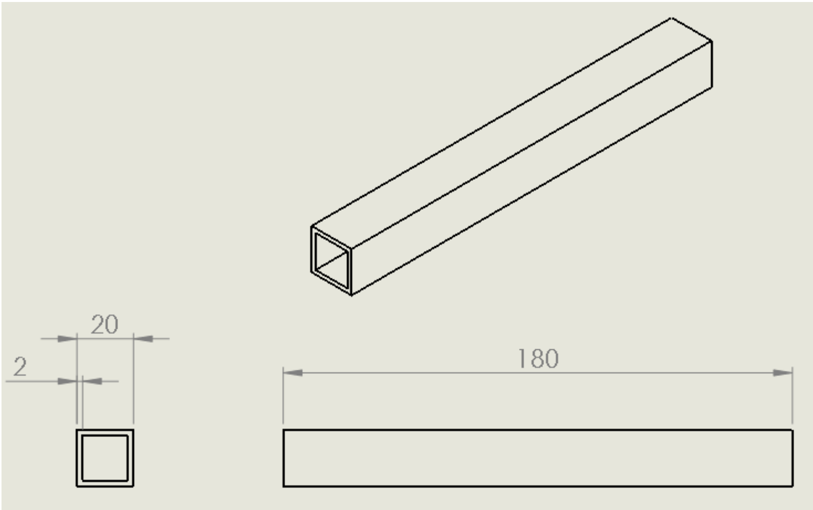

Figure 16: Dimensions of the hollow square beam cross-section (mm)

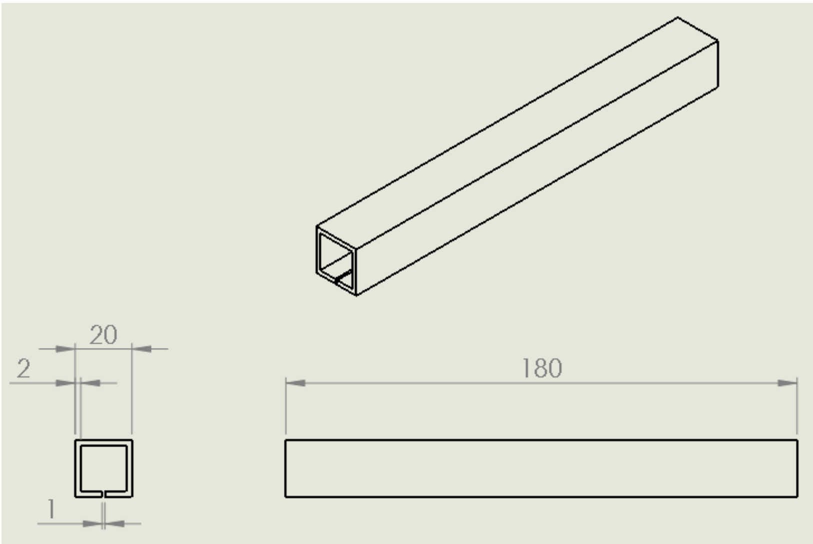

Figure 17: Dimensions of the hollow square beam cross-section with a slit (mm)

Both beams are subjected to the same clockwise torsion. The minor change in the geometry of the beam, created by the presence of the longitudinal slit, results in a different behavior of the beam under torsion. This discontinuity in the cross-sectional shape results in a different shear stress distribution when compared to a continuous cross-section. Such change in the shear stress distribution in turn results in the beam being deformed in a different manner under the applied torsional load. The shear stress distributions for the continuous cross-section beam are shown in Figure 18 and Figure 19 while the shear stress distributions for the beam with a slit are shown in Figure 20, Figure 21 and Figure 22.

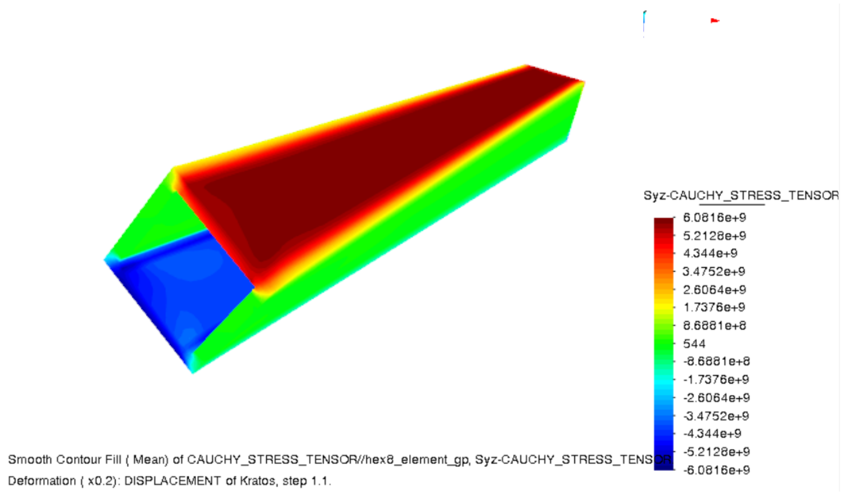

Figure 18: Square beam under torsion shear stress distribution τ_{yz}

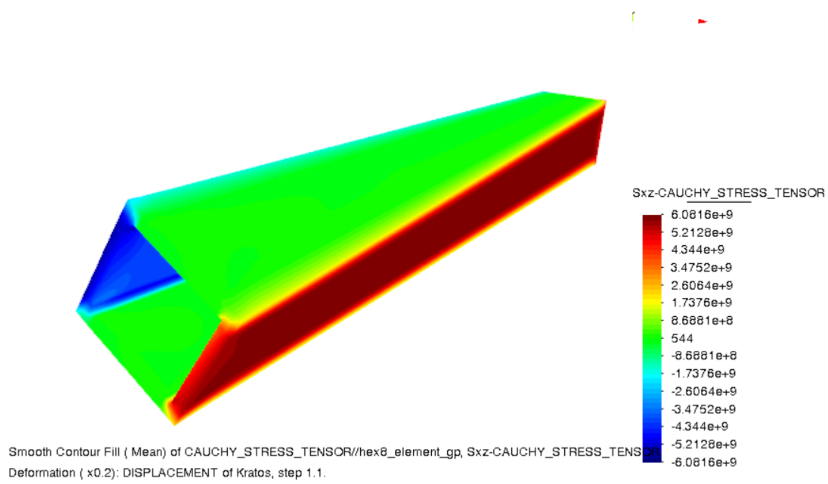

Figure 19: Square beam under torsion shear stress distribution τ_{xz}

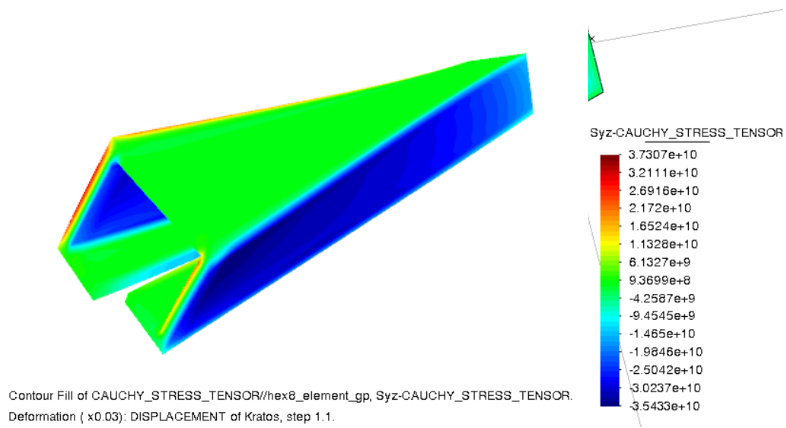

Figure 20: Square beam with a slit under torsion shear stress distribution τ_{yz}

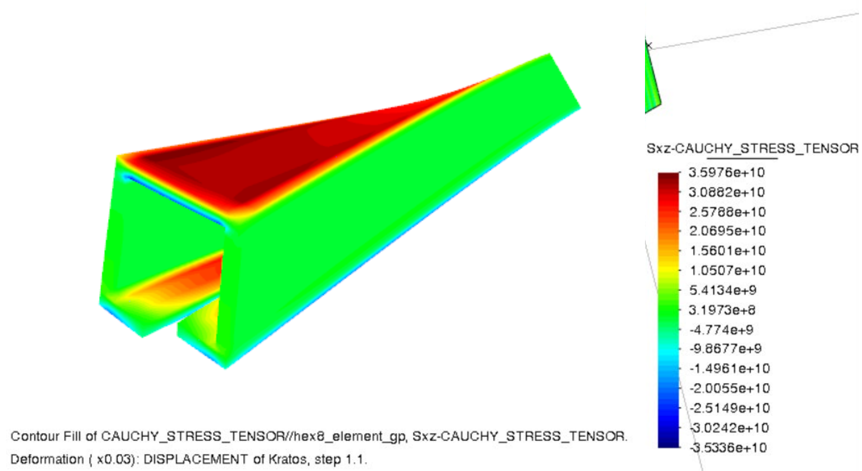

Figure 21: Square beam with a slit under torsion shear stress distribution τ_{xz}

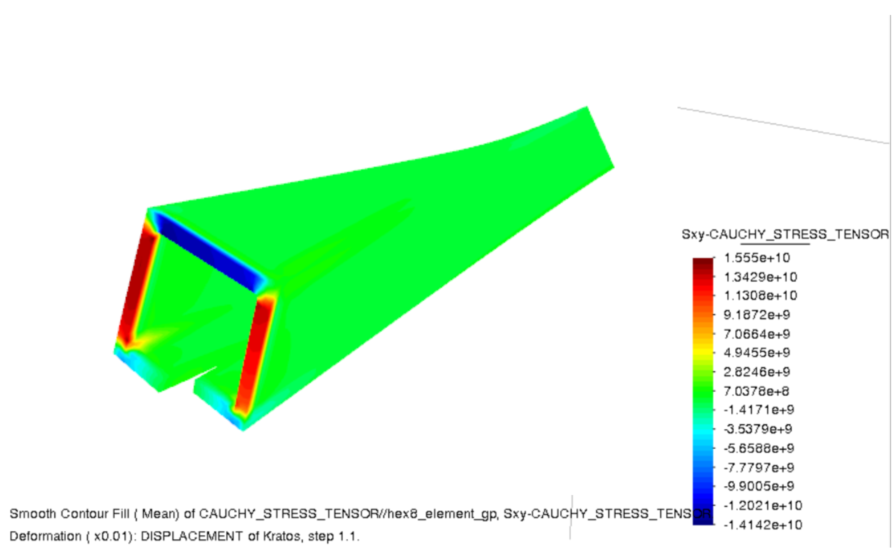

Figure 22: Square beam with a slit under torsion shear stress distribution τ_{xy}

Appendix

Material properties of steel

Young’s modulus E = 2069 MPa

Poisson’s ratio = 0.29

Yield stress = 5.5 MPa