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Finite Elements In Fluid

- As we can see below , the example 1 is solved with Galerkin approach and those depicts that

Pe < 1   solution is stable

Pe>1 solution is unstable

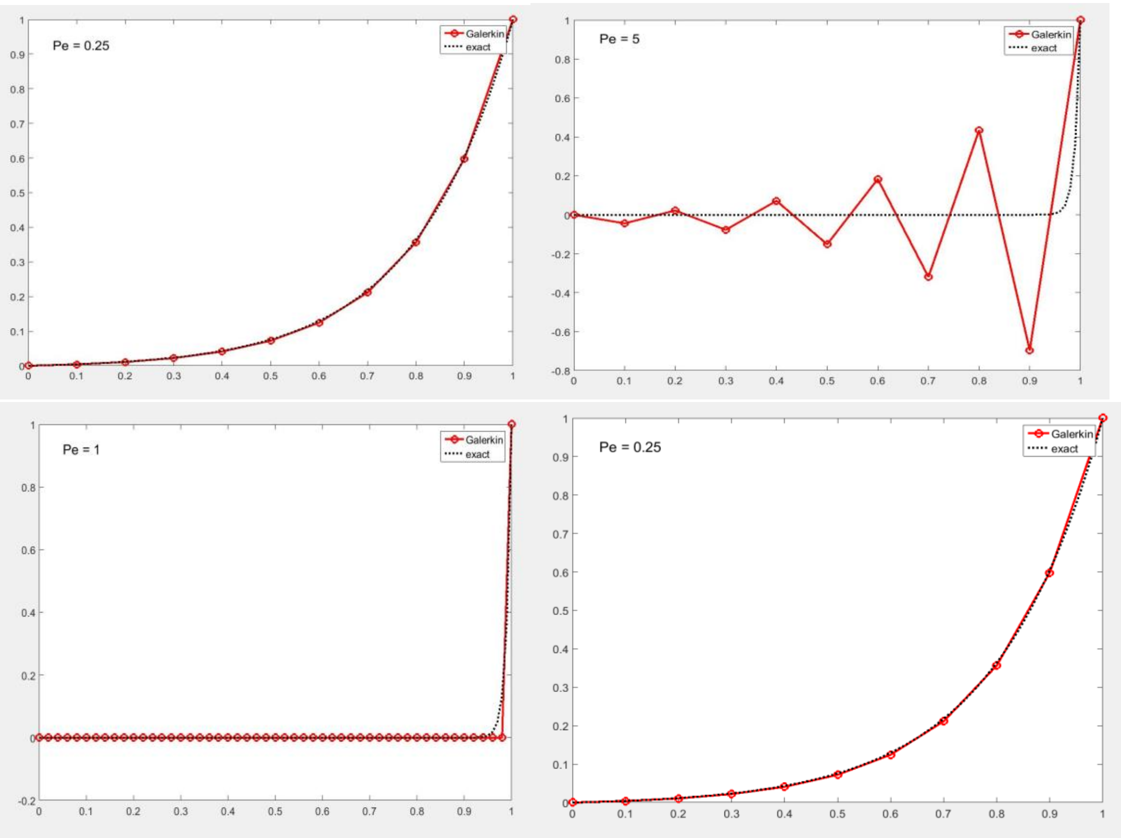

Fig.Galerkin with exm 1

Galerkin method is not stable at higher Pe. No. to stabilize that, stabilization term is added

$$\underbrace{\int_{\Omega}\left[w(\boldsymbol{a}\cdot\nabla u)+\nu\nabla w\cdot\nabla u\right]d\Omega}_{\text{Standard Galerkin}}+\underbrace{\int_{\Omega}\frac{\bar{\nu}}{\|\boldsymbol{a}\|^2}(\boldsymbol{a}\cdot\nabla w)(\boldsymbol{a}\cdot\nabla u)\,d\Omega}_{\text{Added SU term}}=0.$$

The following Matlab code was edited to have SUPG, GLS

For SUPG

```
        h = Xe(end) - Xe(1);
        Ke = zeros(nen);
        fe = zeros(nen,1);
        % Loop on Gauss points
        for ig = 1:ngaus
            N_ig = N(ig,:);
            Nx_ig = Nxi(ig,:)*2/h;
            w_ig = wgp(ig)*h/2;
            Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig))+ w_ig*(tau*a*Nx_ig)'*(a*Nx_ig);
            x = N_ig*Xe; % x-coordinate of the gauss point
            s = SourceTerm(x,example);
            fe = fe + w_ig*(N_ig')*s+w_ig*tau*s*a*Nx_ig';
        end
        % Assmebly
        K(Te,Te) = K(Te,Te) + Ke;
        f(Te) = f(Te) + fe;
    end
```

For GLS

```
        % Loop on Gauss points
        for ig = 1:ngaus
            N_ig = N(ig,:);
            Nx_ig = Nxi(ig,:)*2/h;
            w_ig = wgp(ig)*h/2;
            Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig))+ w_ig*(tau*a*Nx_ig)'*(a*Nx_ig);
            x = N_ig*Xe; % x-coordinate of the gauss point
            s = SourceTerm(x,example);
            fe = fe + w_ig*(N_ig')*s+w_ig*tau*s*a*Nx_ig';
        end
        % Assmebly
        K(Te,Te) = K(Te,Te) + Ke;
        f(Te) = f(Te) + fe;
    end
```

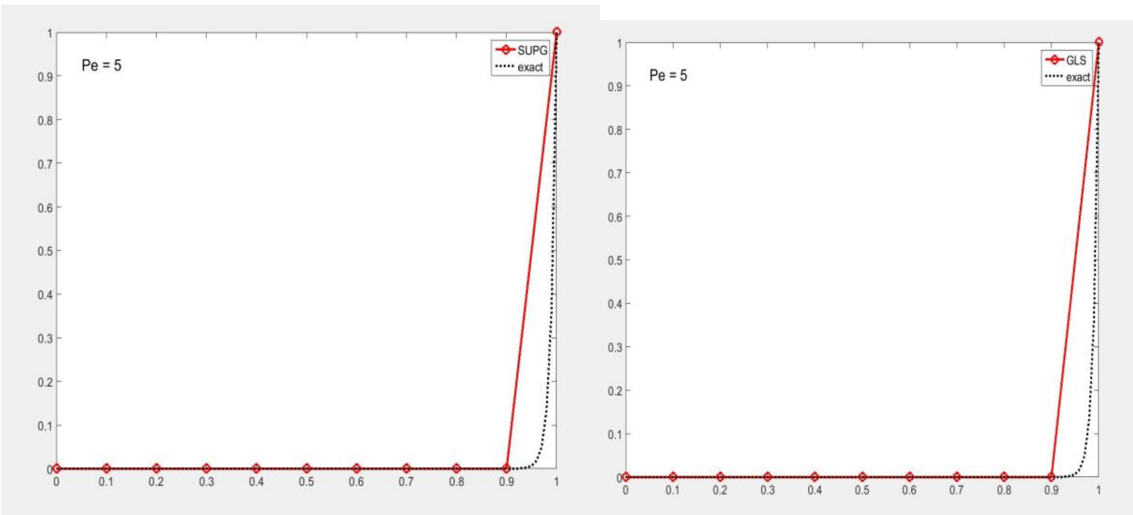

Fig.SUPG and GLS with exm 1

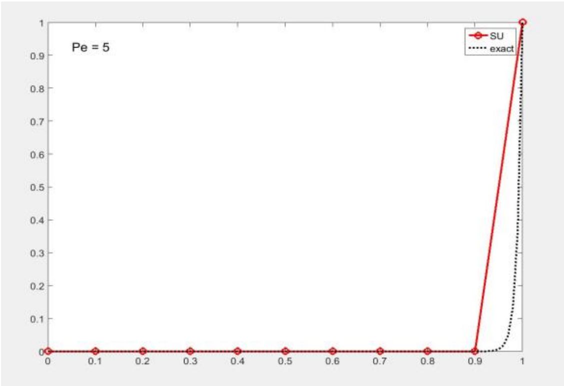

Fig. SU with exm 1

From the above results we can see that, for all for methods results are same, as , the equation is linear so we can say that , stabilization terms reduces the instability due to diffusion equation.