Taylor Galerkin Method (2 steps)

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Considering

$$u^{\epsilon}_t + f_x(u^{\epsilon}) = \varepsilon u^{\epsilon}_{xx}$$

And the second order expansion:

$$u^{\epsilon n+1} \approx u^{\epsilon n} + \Delta t \underbrace{\left(u^{\epsilon n} + \frac{\Delta t}{2} u^{\epsilon n}_t\right)_t}_{u^{\epsilon n+1/2}}$$

we have:

$$\begin{cases} u^{\epsilon n+1/2} = u^{\epsilon n} + \frac{\Delta t}{2} u^{\epsilon n}_t \\ u^{\epsilon n+1} = u^{\epsilon n} + \Delta t\, u^{\epsilon n+1/2}_t \end{cases} \Longrightarrow \begin{cases} u^{\epsilon n+1/2} = u^{\epsilon n} + \frac{\Delta t}{2}\left(\varepsilon u^{\epsilon n}_{xx} - f_x(u^{\epsilon n})\right)_t \\ u^{\epsilon n+1} = u^{\epsilon n} + \Delta t\left(\varepsilon u^{\epsilon n+1/2}_{xx} - f_x(u^{\epsilon n+1/2})\right) \end{cases}$$

$$\frac{\Delta u^{\epsilon}}{\Delta t} = \varepsilon u^{\epsilon n+1/2}_{xx} - f_x(u^{\epsilon n+1/2})$$

$$\left(w, \frac{\Delta u^{\epsilon}}{\Delta t}\right) = \left(w, \varepsilon u^{\epsilon n+1/2}_{xx}\right) - \left(w, f_x(u^{\epsilon n+1/2})\right)$$

$$= \left(nw, \varepsilon u^{\epsilon n+1/2}_x\right)_\Gamma - \left(w_x, \varepsilon u^{n+1/2}_x\right) + \left(w_x, f(u^{\epsilon n+1/2})\right) - \left((wn, f(u^{\epsilon n+1/2})\right)_\Gamma$$