Unsteady Transport Problem

Lei Pan

Leap-frog method is a second-order accuracy method while TG3-2S is a third-order method. The stable condition for Leap-frog method is $C^2 < 0.57$ and for TG3-2S method it is $C^2 < 0.75$. And in our example, we have chosen $\mathrm{C} = 0.5$ which makes both methods stable.

In below cases of applications of two methods, we will see that the ξ amount can influence the accuracy of the results. And higher order method can considerably increase the accuracy of our solutions.

1. Problem 1

1) Leap-frog Method

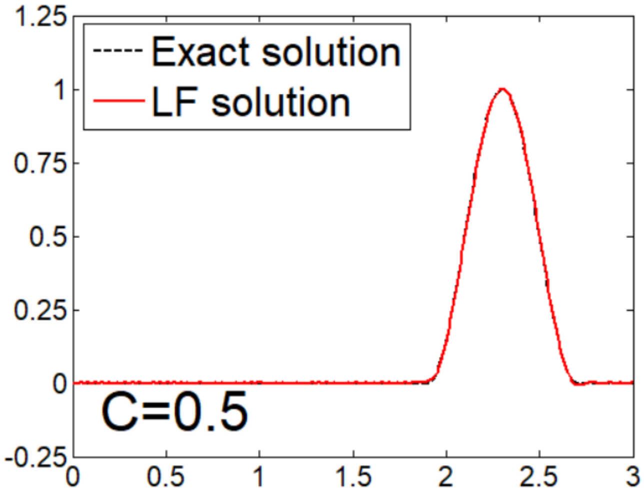

Figure 1. Problem 1 result of LF method

2) TG3-2S

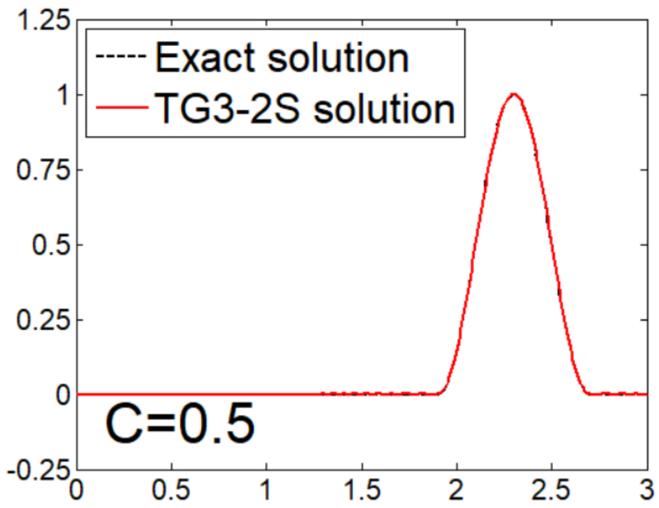

Figure 2. Problem 1 result of TG3-2S method

Higher order method can provide more accurate result. But in problem 1, the ξ is small enough so that the LF method also works as well as TG3-2S does. They are both accurate.

2. Problem 2

1) Leap-frog Method

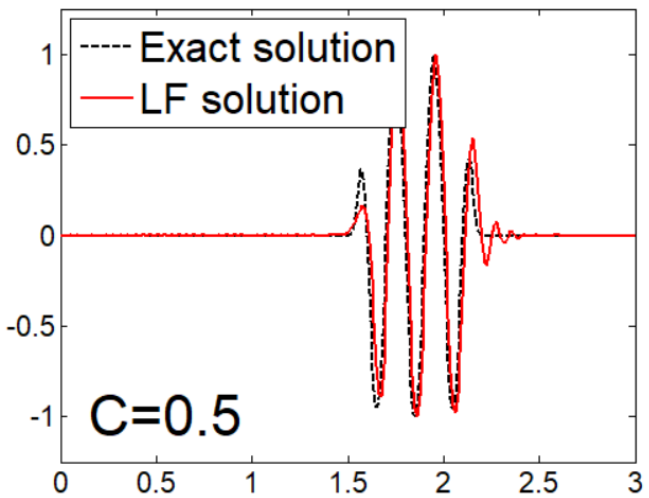

Figure 3. Problem 2 result of LF method

2) TG3-2S Method

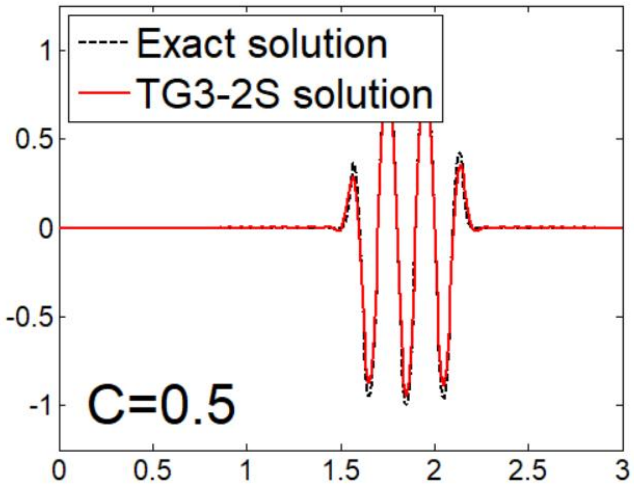

Figure 4. Problem 2 result of TG3-2S method

When ξ is big, the higher order method has presented its advantage. In above results, even though LF method's result still get close to the exact solution, but it doesn't work as well as it has done in problem 1. On the one hand, the TG3-2S method's result is more accurate than LF method. On the other hand, the TG3-2S method is also influenced by the increasing of ξ. The accuracy will decrease when ξ increases and problem 3 will show this case.

3. Problem 3

1) Leap-frog Method

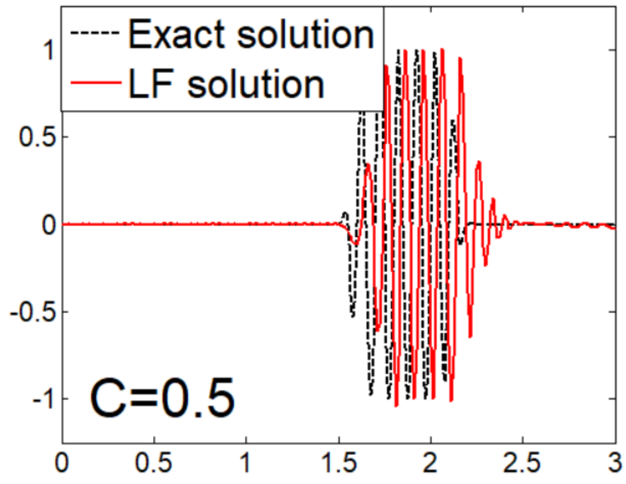

Figure 5. Problem 3 result of Leap-frog Method

2) TG3-2S Method

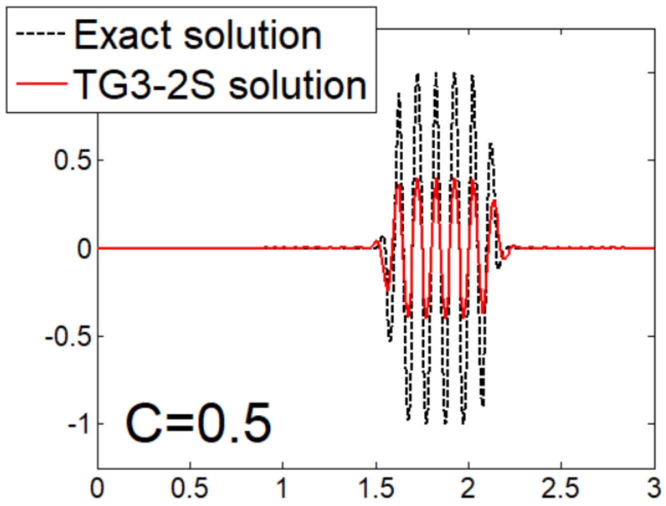

Figure 6. Problem 3 result of TG3-2S Method

In problem 3, the ξ is double the amount of itself in problem 2, LF method becomes totally inaccurate in the oscillation parts while TG3-2S doesn't work well in the wave peaks parts. In order to solve this problem, we need to keep increasing the order like applying FLTG method which is fourth order method.

4. Problem 4

1) Leap-frog Method

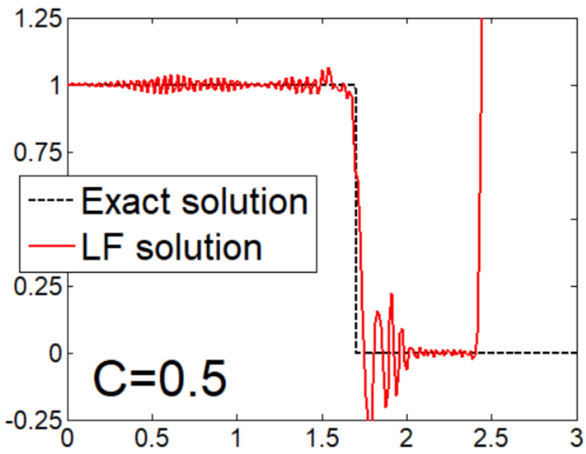

Figure 7. Problem 4 result of Leap-frog Method

2) TG3-2S Method

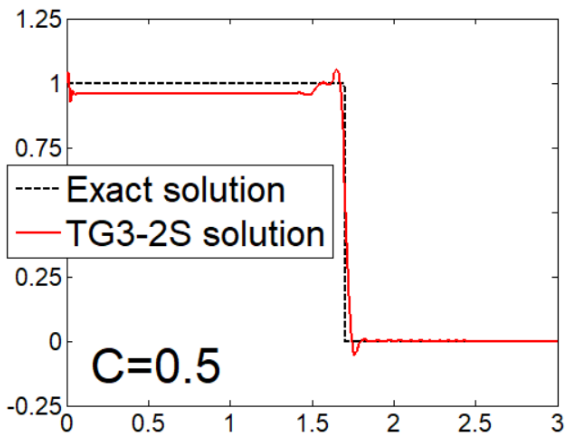

Figure 8. Problem 4 result of TG3-2S Method

In problem 4, the Leap-frog Method's result is inaccurate and oscillation happens on the whole domain. For TG3-2S method, the result oscillates in the beginning and the area near the vertical line. And on the left area of vertical line whose initial boundary conditions is $u_0 = 1$, TG3-2S is stable but not accurate. On the right area whose initial boundary conditions is $u_0 = 0$, TG3-2S works well.

5. Modifying the Code

1) Leap-frog Method

```
case 5 % LF
    A = M;
    B =  -2*a*dt*C;
    methodName = 'LF';
```

Figure 8. Leap-frog Method code in system.m

```
for n = 1:nStep
    if method==5
        if n==1  %because the TG3-2S is a two steps method
            [A,B,methodName] = System(1,M,K,C,a,dt); %the first step
            A = A(ind_unk,ind_unk);%should be computed by other method
            B = B(ind_unk,ind_unk);
            Du = A\(B*u(ind_unk,n) + f);
            u(ind_unk,n+1) = u(ind_unk,n) + Du;
        else
            [A,B,methodName] = System(5,M,K,C,a,dt);
            A = A(ind_unk,ind_unk);
            B = B(ind_unk,ind_unk);
            Du = A\(B*u(ind_unk,n) + f);
             u(ind_unk,n+1) = u(ind_unk,n-1) + Du;
        end
    else
    Du = A\(B*u(ind_unk,n) + f);
    u(ind_unk,n+1) = u(ind_unk,n) + Du;
    end
end
```

Figure 9. Modifications for Leap-frog Method code in main.m

2) TG3-2S Method

```
case 6 % TG3-2S
    A = M;
     B =  -a*dt*C-1/2*dt^2*a^2*(K-1/3*dt*a*(M^-1)*C*K-1/9*dt^2*a^2*(M^-1)*K*K);
    methodName = 'TG3-2S';
```

Figure 8. TG3-2S Method code in system.m