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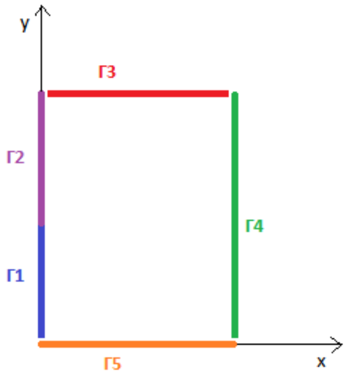
Exercise 3

Domain $\Omega = (0,2) \times (0,3)$, boundary conditions $\Gamma_1 = (0,0) \times \left(0, \frac{3}{2}\right), \Gamma_2 = (0,0) \times \left(\frac{3}{2}, 3\right), \Gamma_3 = (0,2) \times (3,3), \Gamma_4 = (2,2) \times (0,3), \Gamma_5 = (0,2) \times (0,0)$.



$$v = 0 \;\; in \;\; \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_5 \quad v_y = -1 \; in \; \Gamma_4$$

a) The Stokes problem

$$-\vartheta \Delta \boldsymbol{v} + \nabla p = \boldsymbol{f} \; in \; \Omega$$
$$\nabla \cdot \boldsymbol{v} = 0 \; in \; \Omega$$
$$\boldsymbol{v} = \boldsymbol{0} \; on \; \partial\Omega$$

Oil viscosity varies in a wide range. Let us accept the following value ϑ = 1.98.
The weak form of the problem takes the view:

$$a(w,v) + b(w,p) + b(v,q) = (w,f) + (w,t)_{\Gamma_N} \quad \forall\, (w,q) \in V \times Q$$

Where $a(w,v) = \int_\Omega \nabla w : \vartheta \nabla v \, d\Omega$ and $b(v,q) = -\int_\Omega q \nabla \cdot v \, d\Omega$, t is the step.

After discretization with Galerkin formulation the Stokes problem takes the form:

$$\begin{pmatrix} K & G \\ G^T & 0 \end{pmatrix} \begin{pmatrix} v \\ p \end{pmatrix} = \begin{pmatrix} b \\ h \end{pmatrix}$$

Where K=$\boldsymbol{A}^e K^e$ is the matrix consisting of elements $K_{ab}^e = \int_{\Omega^e} B_a^T C_\vartheta B_b \, d\Omega$ with

$$B_a^T = \begin{pmatrix} \frac{\partial N_a}{\partial x_1} & \frac{\partial N_a}{\partial x_2} & \frac{\partial N_a}{\partial x_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_a}{\partial x_1} & \frac{\partial N_a}{\partial x_2} & \frac{\partial N_a}{\partial x_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\partial N_a}{\partial x_1} & \frac{\partial N_a}{\partial x_2} & \frac{\partial N_a}{\partial x_3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

G=$\boldsymbol{A}^e G^e$ is a matrix consisting of elements

$$G_{r\hat{a}}^e = b\left(N_a e_i, \widehat{N}_{\hat{a}}\right)_{\Omega^e}$$

b=$\boldsymbol{A}^e b^e$ is a vector which consists of elements

$$b_r^e = \left(N_a, f_i^h\right)_{\Omega^e} + \left(N_a, t_i^{hi}\right)_{\Gamma_N{}^e} - \sum_q K_{rs}^e \, v_{Ds}^e$$

h=$\boldsymbol{A}^e h^e$ is a vector which consists of elements

$$b_{\hat{a}}^{e} = \sum_{q} \left(\widehat{N}_{\hat{a}}, \nabla \cdot (N_a e_i)\right)_{\Omega^e} v_{Dr}^{e}$$

Here $v_{Ds}^{e} = v_{Djb}^{e}$ if v_{Dj}^{e} is prescribed at node f, and equals to zero otherwise.

For quadrilateral and triangular elements 10x10 mesh size is chosen as with this size the solution is more precise and has less fluctuations. With bigger number elements the solution remains the same as it can be seen from the following figures 1-3.

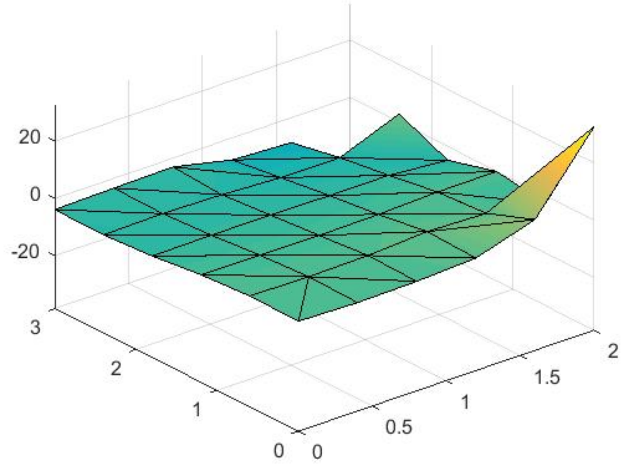


Fig. 1: pressure, triangular elements, number of elements in each direction 5

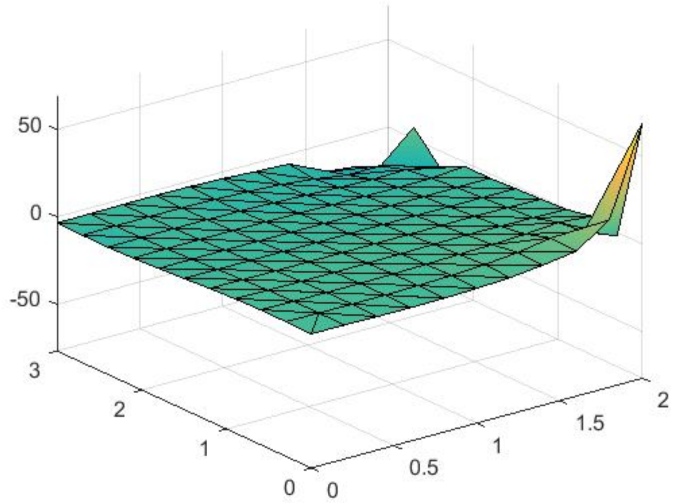


Fig. 2: pressure, triangular elements, number of elements in each direction 10

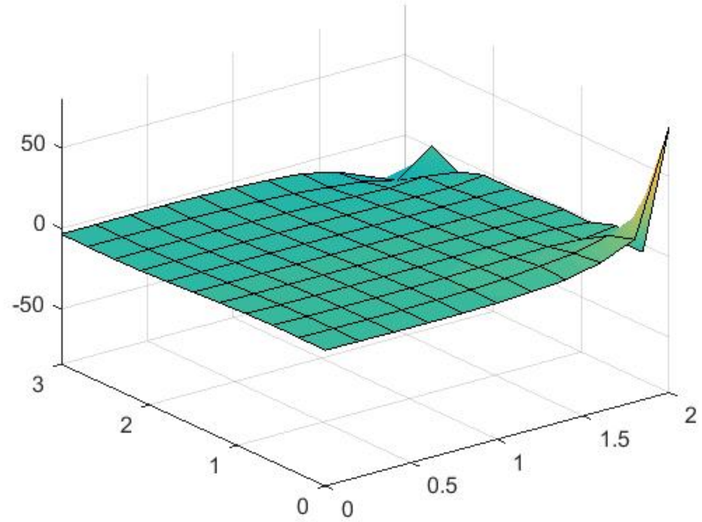


Fig. 3: pressure, quadrilateral elements, number of elements in each direction 10

At the same time for triangular elements with bubble function, wild fluctuations are observed in case of 5x5 mesh. This problem is solved with the 10x10 mesh however slight fluctuations remain on the border. Using 15x15 mesh, we receive more accurate solution.

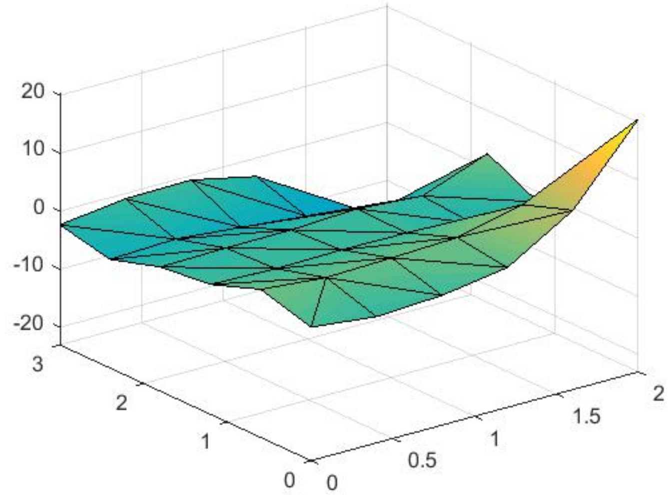


Fig. 4: pressure, triangular elements with bubble function, number of elements in each direction 5

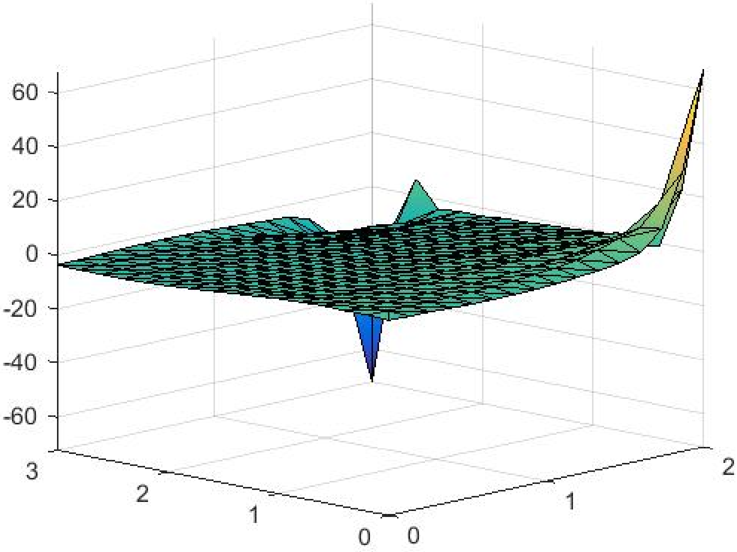


Fig. 5: pressure, triangular elements with bubble function, number of elements in each direction 15

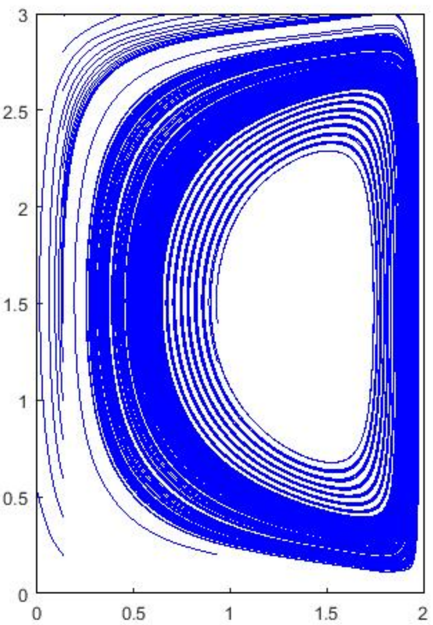


Fig. 6: streamlines, triangular elements with bubble function, number of elements in each direction 15

As it can be seen from the figures above, the development of secondary vortex in the right bottom corner of cavity becomes progressively apparent and a third vortex appears at the lower left corner.

b) Let us consider Navier-Stokes equation

$$-\vartheta\nabla^2\boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \nabla p = \boldsymbol{f} \ in\ \Omega$$

$$\nabla\cdot\boldsymbol{v} = 0 \ in\ \Omega$$

$$\boldsymbol{v} = \boldsymbol{0} \ on\ \partial\Omega$$

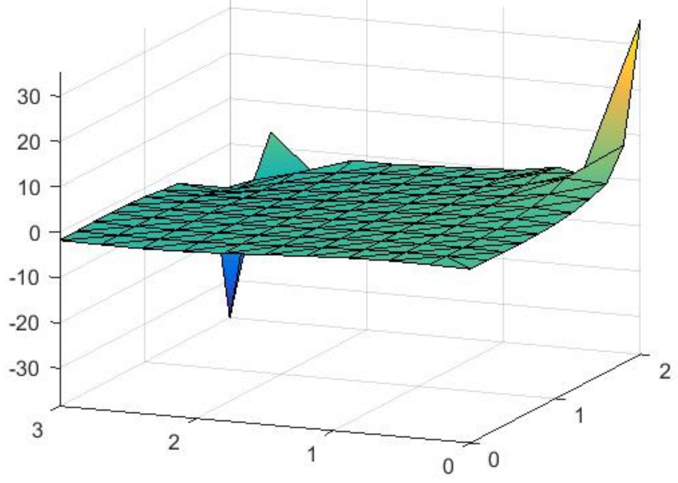


Fig. 7: pressure, triangular elements, number of elements in each direction 10, Re=1

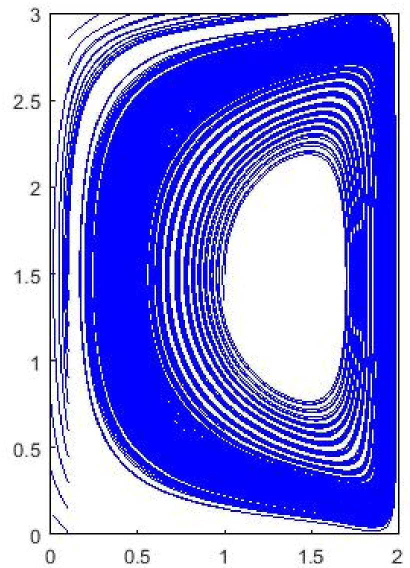


Fig. 8: streamlines, triangular elements, number of elements in each direction 10, Re=1

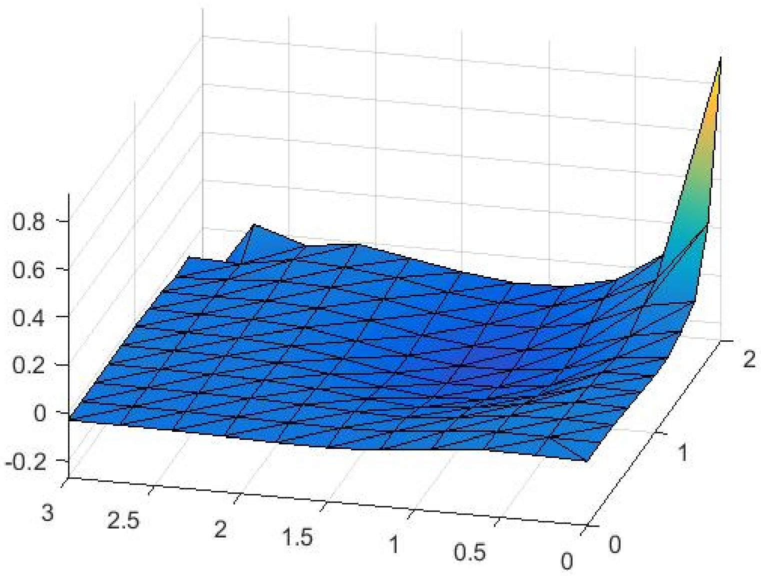


Fig. 9: pressure, triangular elements, number of elements in each direction 10, Re=100

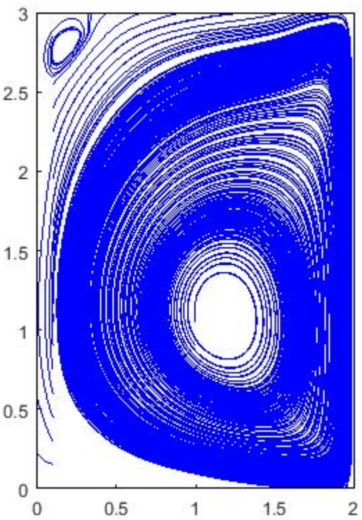


Fig. 10: streamlines, triangular elements, number of elements in each direction 10, Re=100

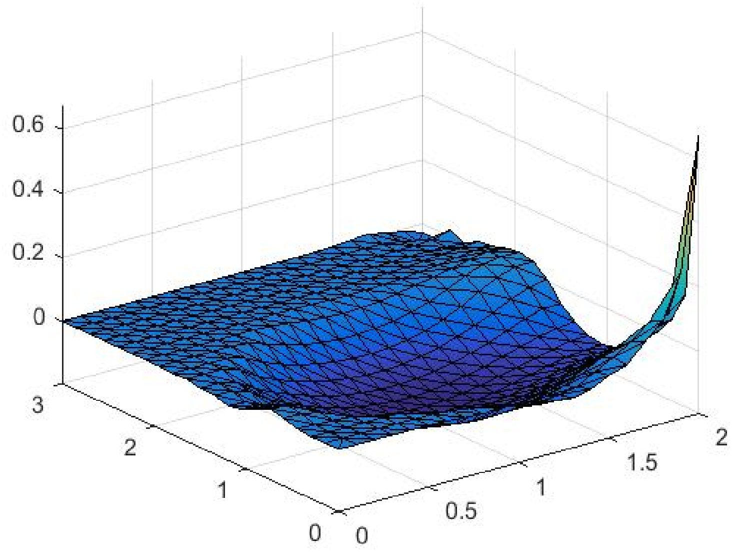


Fig. 11: pressure, triangular elements, number of elements in each direction 20, Re=1000

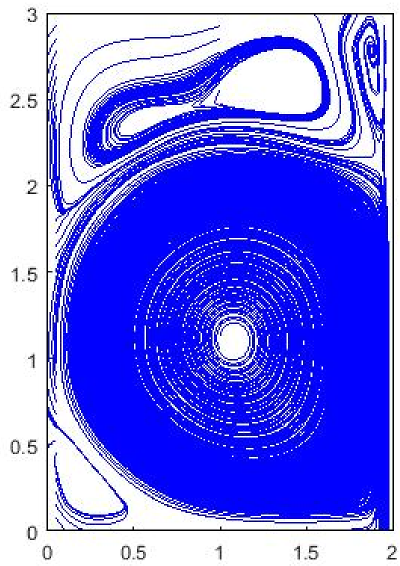


Fig. 12: streamlines, triangular elements, number of elements in each direction 20, Re=1000

As in previous task, for Reynolds number 1 and 100 10x10 mesh was used. However, for Re=1000 solution does not converge with this size of the mesh. In this case 20x20 mesh was considered. As it can be seen from figures 11 and 12, the solution converges in this case. All solutions are considered using 100 iterations. The solution also can be improved with the increasing of iterations. For Re=2000 even for 25x25 mesh solution has wild oscillations.

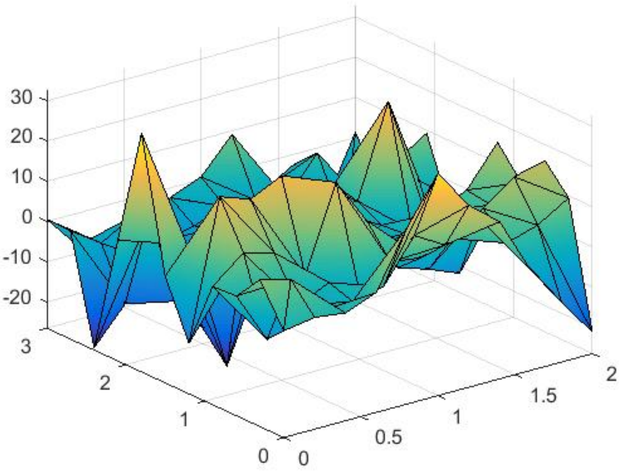


Fig. 13: pressure, triangular elements, number of elements in each direction 10, Re=2000

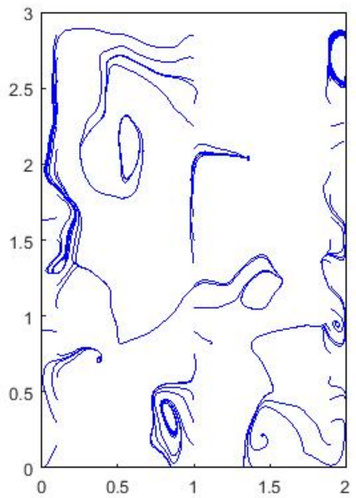


Fig. 14: streamlines, triangular elements, number of elements in each direction 10, Re=2000

As it can be seen from figures 13 and 14, where considered the number of iterations 1000, for Re=2000 solution still remains unstable. Even for 5000 iterations the solution does not converge.

As can be seen from figures 7-12 the position of main vortex moves towards the center of the cavity when the Reynolds number increases. The development of secondary vortex in the right bottom corner of cavity becomes progressively apparent and a third vortex appears at the lower left corner.