

UNIVERSITAT POLYTECHNICA DE CATALUNYA
MSC COMPUTATIONAL MECHANICS
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FEF Matlab

REPORT 1

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1 Galerkin's Method Preliminaries

In the first part of this report, we will run the first example in the Matlab code with 4 different sets of values for the convection coefficient a , the diffusion coefficient ν , and number of linear elements. These results are shown below.

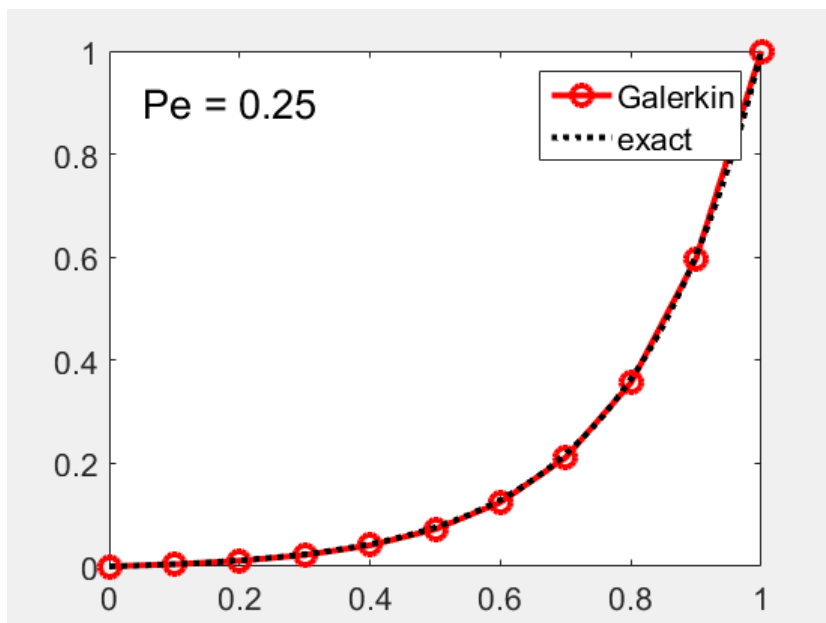


Figure 1: Results for 10 elements, $a = 1$, $\nu = 0.2$

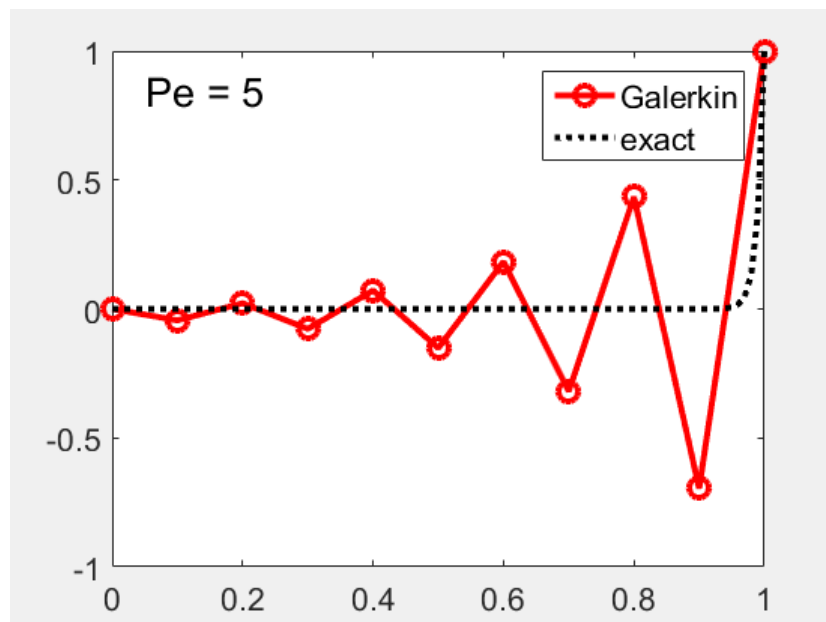


Figure 2: Results for 10 elements, $a = 20$, $\nu = 0.2$

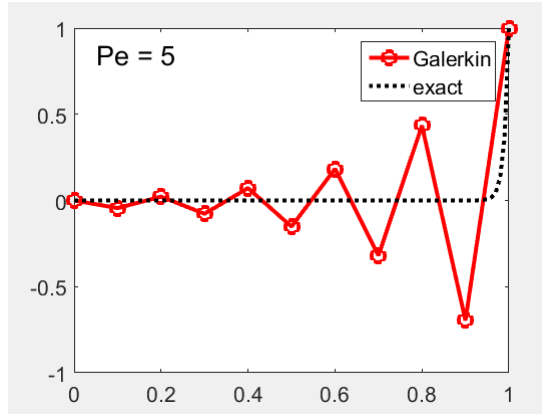


Figure 3: Results for 10 elements, $a = 1$, $\nu = 0.01$

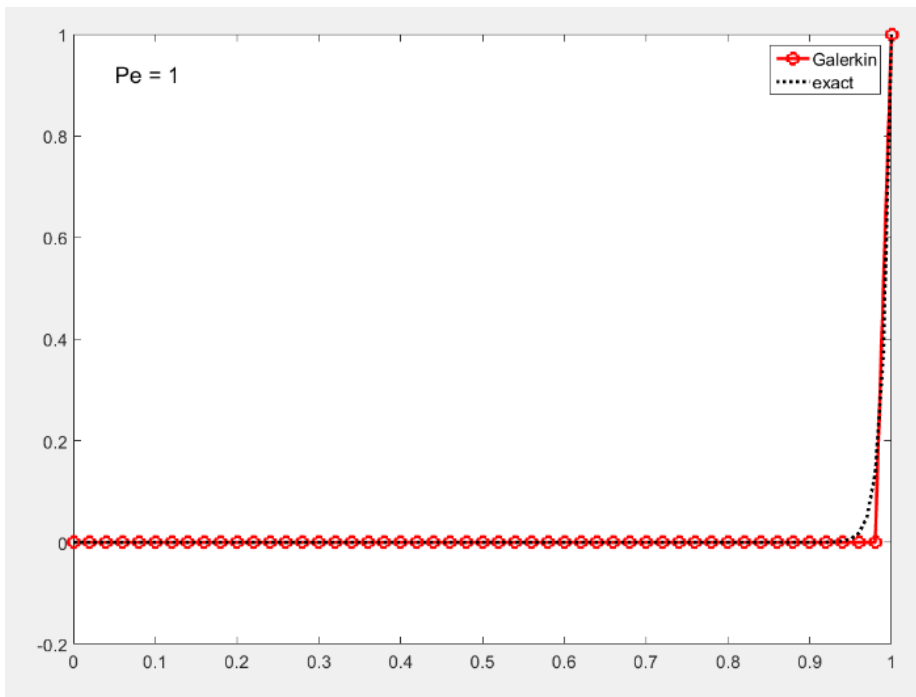


Figure 4: Results for 50 elements, $a = 1$, $\nu = 0.01$

From these previous figures we can confirm that when the Peclet number is over 1, The graphs exhibit oscillatory behavior. This is a result of only using galerkins approximation with no stabilization techniques.

2 Solving with SU, SUPG, and GLS Stabilizations

We will now solve the third example in the Matlab code using **SU**, **SUPG**, and **GLS** Stabilizations. This can be seen in the figures below.

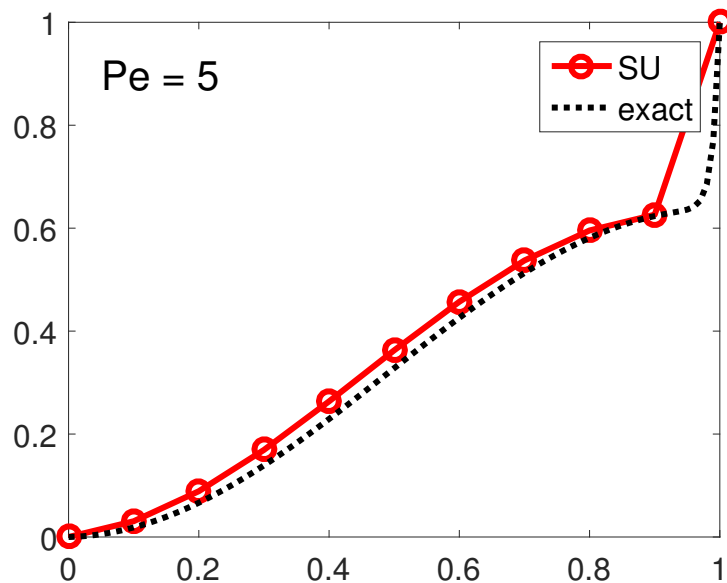


Figure 5: **SU** Results for 10 elements, $a = 1$, $\nu = 0.01$

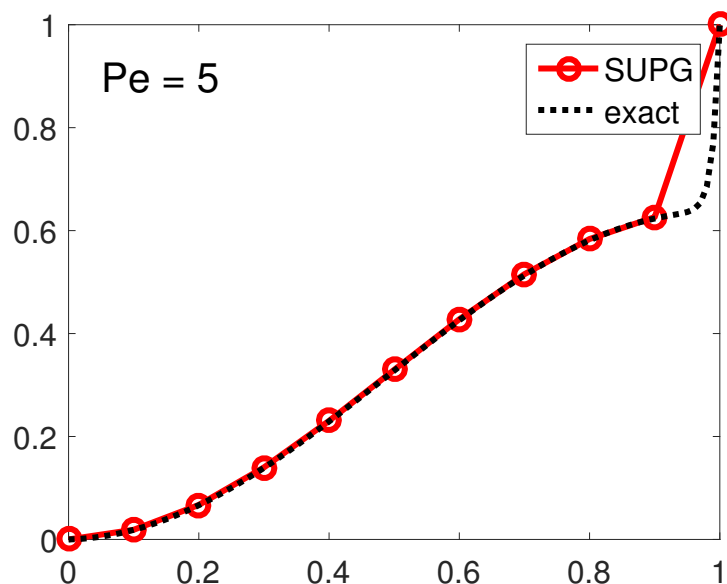


Figure 6: **SUPG** Results for 10 elements, $a = 1$, $\nu = 0.01$

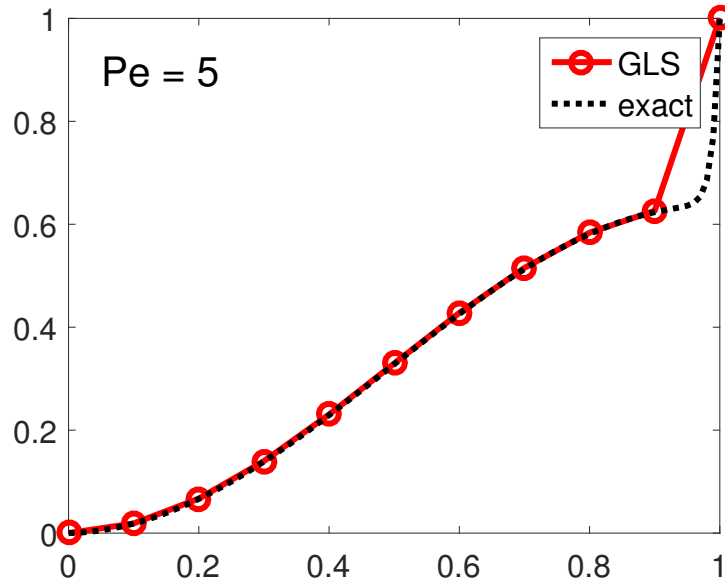


Figure 7: **GLS** Results for 10 elements, $a = 1$, $\nu = 0.01$

It is important to note that with the given problem parameters, the SU method did not achieve the exact solution, while the GLS and SUPG methods gave the exact solution and produced identical results. The graphs above confirm this. The relevant code modifications are shown below.

```

Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig)) + w_ig*tau*((a*Nx_ig)'*(a*Nx_ig))
x = N_ig*Xe; % x-coordinate of the gauss point
s = SourceTerm(x,example);
fe = fe + w_ig*(N_ig')*s +

```

%Adding the relevant terms of SUPG method to stiffness matrix
 %Adding the relevant terms of SUPG method to force side equation

Figure 8: **SUPG** Code Modifications

```

Ke = Ke + w_ig*(N_ig'*(a*Nx_ig) + Nx_ig'*(nu*Nx_ig)) + w_ig*tau*((a*Nx_ig)'*(a*Nx_ig))
x = N_ig*Xe; % x-coordinate of the gauss point
s = SourceTerm(x,example);
fe = fe + w_ig*(N_ig')*s +

```

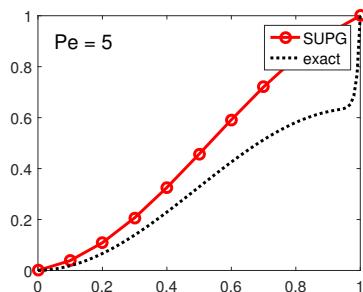
%Adding the relevant terms of GLS method to stiffness matrix
 %Adding the relevant terms of GLS method to force side equation

Figure 9: **GLS** Code Modifications

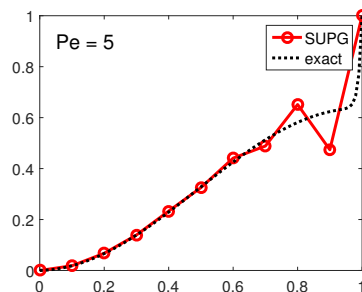
As seen above, the convection terms have been added to both the force and stiffness sides of the equation. Since we are working on a one dimensional linear element, the diffusion stabilization terms can be neglected. It is also worth noting that even though there is a high Peclet number of 5 for these cases, there is no oscillatory behaviors because of the stabilization techniques employed.

3 Effects of Stabilization Parameter

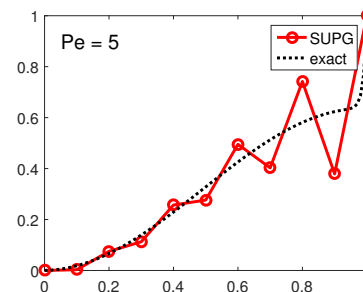
We will now test the effects of the stabilization parameter τ by testing SUPG methods with $\tau=0$, $\tau=0.01$, and $\tau=1$. These results can be seen below.



Stabilization with $\tau = 1$



Stabilization with $\tau = 0.01$



Stabilization with $\tau = 0$

Here we can see that manually setting τ equal to zero eliminates stabilization from the equation and produces Galerkin results with oscillatory behavior since the Peclet number is 5. Setting $\tau=0.01$ reduces the amplitude of these oscillations towards zero and gives better results but we still see some oscillations as the graph gets close to one.