Finite Element in Fluids

Un Steady

Navier Stokes Equation

Home Work -9

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MS-Computational Mechanics

Problem

Un-Steady NS Equation

$$
\begin{aligned}
\boldsymbol{v}_t + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} - \nu\nabla^2\boldsymbol{v} + \boldsymbol{\nabla}p &= \boldsymbol{f} && \text{in } \Omega,\\
\boldsymbol{\nabla}\cdot\boldsymbol{v} &= 0 && \text{in } \Omega,\\
\boldsymbol{v} &= \boldsymbol{v}_D && \text{on } \Gamma_D,\\
\boldsymbol{n}\cdot\boldsymbol{\sigma} &= \boldsymbol{t} && \text{on } \Gamma_N,
\end{aligned}
$$

In this problem a transient incompressible NS flow through a cavity is simulated. The Galerkin formulation is used for spatial discretization while time is descritized through theta method or by Chorin Temam projection method.

- Chorine-Temam Projection method
- Semi Implicit first order monolitic method
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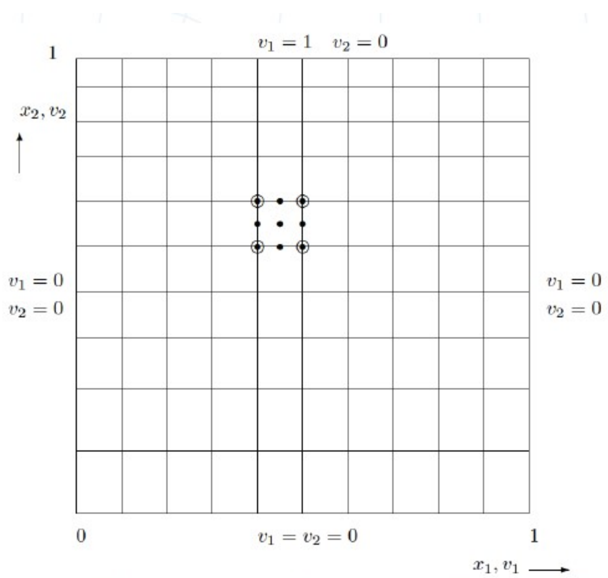
Problem description

It's a standard benchmark flow through cavity problem in square domain of 1 x 1 where velocity is zero at three boundaries (no slip condition) and has a value of 1 in x direction at cavity open edge.



For Re =100, (Reynold Number) The two dimensional problem in the square domain Ω =]0, 1[x]0, 1[, with boundary conditions can be seen in above diagram. It poses a close solution with the velocity field v = (v1,v2) and pressure field p.

The problem is descritized with 10 elements in each direction.

a) Chorin- Temam Projection Method

It's a 2 steps method in which velocity and pressure are calculated separately through intermediate velocity. In first step we solved transient convection- diffusion term with boundary conditions.

$$M u^{i+1} + \Delta t G p^{i+1} = M u^{*}$$
$$G^{T} u^{i+1} = 0$$

In 2nd step, we have linear system of equation to compute pressure in such a way;

$$\begin{bmatrix} M & \Delta t G \\ G^{T} & 0 \end{bmatrix} \begin{bmatrix} u^{i+1} \\ p^{i+1} \end{bmatrix} = \begin{bmatrix} M u^{*} \\ 0 \end{bmatrix}$$

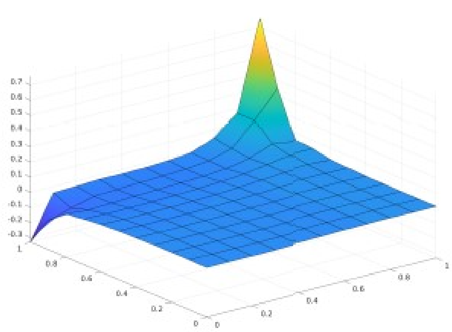
b) Semi- Implicit First Order Monolithic Method

The spatial discretization is done by Galerkin formulation. While In this scheme, time discretization is done through theta method

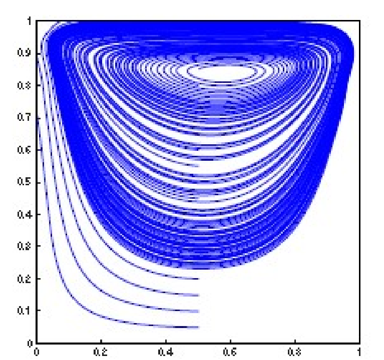
$$\begin{bmatrix} M + \Delta t\theta (K + C) & \Delta t\theta G \\ G^{T} & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta p \end{bmatrix} = \begin{bmatrix} \Delta t (F - [K + C] u^{i} - G p^{i}) \\ 0 \end{bmatrix}$$

Results

The domain is meshed by 10 x 10 Q2Q1 element types with Re = 100, with time step of 0.01 and total time of 0.1 sec.

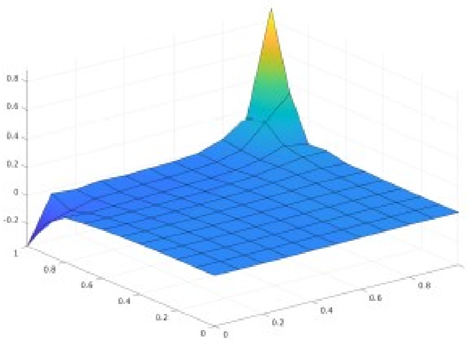


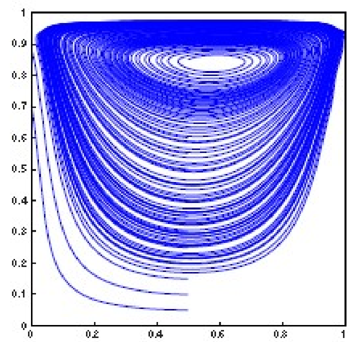
Pressure Field



Streamlines at t = 0.5 sec

Figure-1: Response of Q2Q1 element in Chorin- Temam projection Method





Pressure Field Streamlines at t = 0.5 sec

Figure-2: Response of Q2Q1 element in Semi- Implicit Monolitic Method

Comments

It is evident that both methods shows more or less similar results while semi implicit monolithic method is a little bit faster than Chorin Temam projection method.