Master Of Science in Computational Mechanics

Finite Elements in Fluid

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Assignment 6

(Stokes and Navier-Stokes Numerical examples)

1 Stokes problem:

$$\begin{aligned} -\nu\nabla^2 v + \nabla p = b \quad & in\Omega \\ \nabla \cdot v = 0 \quad & in\Omega \end{aligned}$$

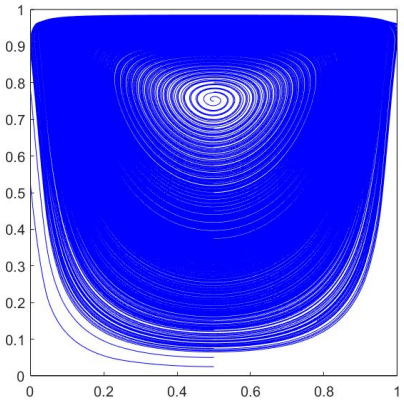
Weak form,

$$\begin{cases} \int_\Omega \nabla w : \nu\nabla v d\Omega - \int_\Omega p\nabla \cdot w d\Omega = \int_\Omega w \cdot b d\Omega & \forall w \in \nu \\ \int_\Omega q\nabla \cdot v d\Omega = 0 & \forall q \in Q \end{cases}$$

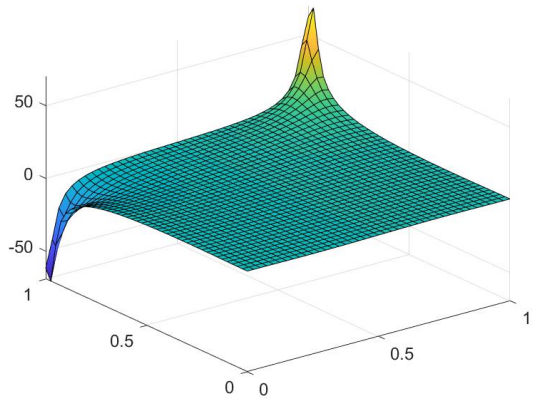
Galerkin descretization:

$$\begin{pmatrix} K & G \\ G^T & 0 \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} f \\ h \end{pmatrix}$$

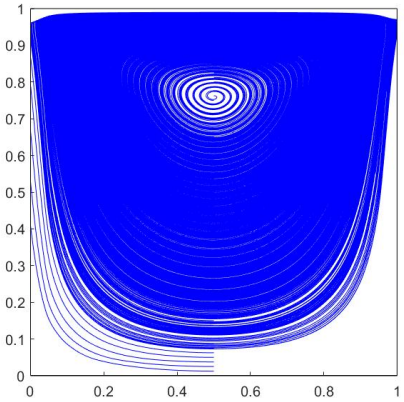
The problem was solved with 40 elements in both x and y directions. following results were obtained for different elements for fist order and quadratic pressure and velocity



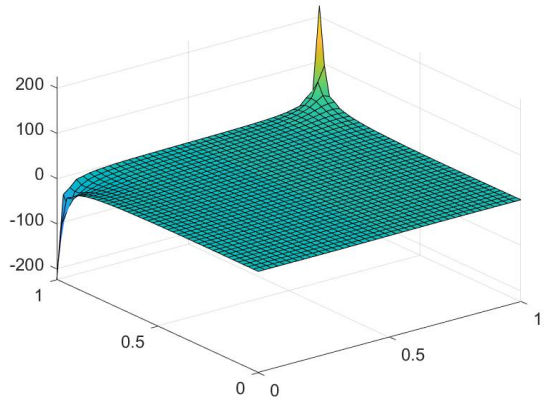
(a) quadrilaterals-Q1Q1



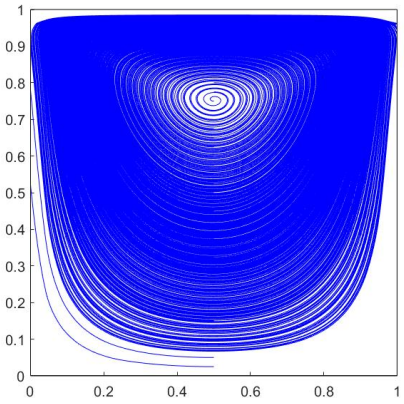
(b) quadrilaterals-Q1Q1



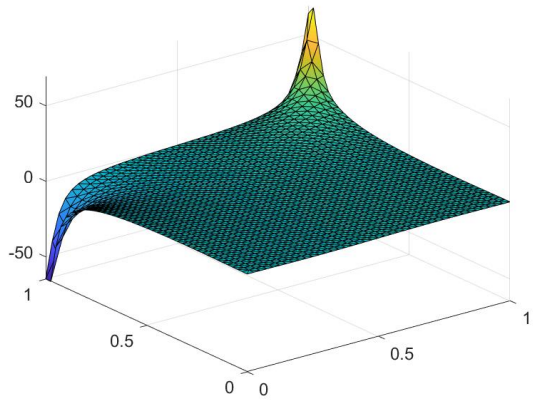
(c) quadrilaterals-Q2Q1



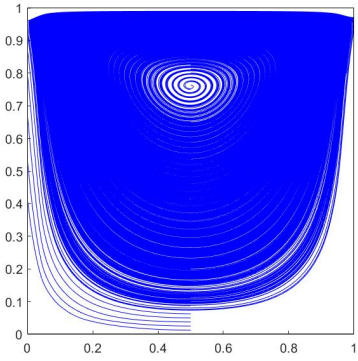
(d) quadrilaterals-Q2Q1



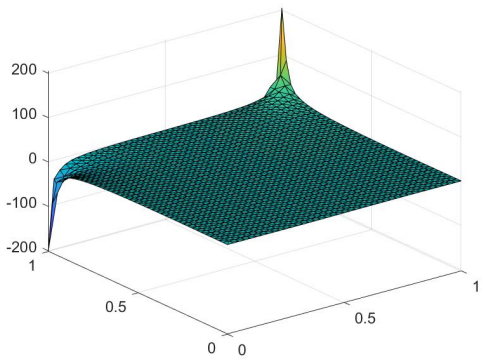
(e) triangles-P1P1.



(f) triangles-P1P1.



(g) triangles-P2P1.

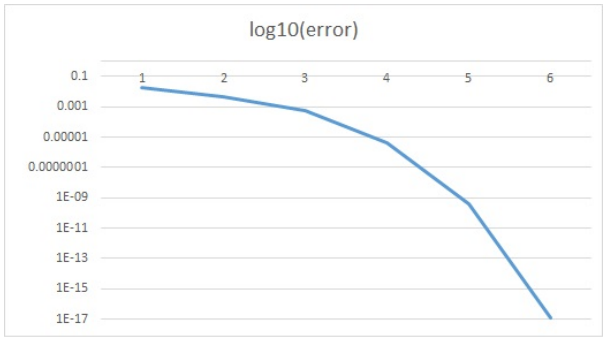


(h) triangles-P2P1.

2 Navier-Stokes problem:

The given code was modified for solving the Navier-Stokes problem using Piccard's method and Newton-Raphson method. Figure below show the convergence curve for both methods. It can be seen that for Piccard's method it takes 13 iterations to solve as it converges linearly while for Newton-Raphson it solves with just 5 iteration as the convergence takes place is quadratic. 10 quadrilaterals-Q2Q1 elements were considered in each direction, tolerance considered was $0.5E-08$.

(a) Piccard's method



(b) Newton-Raphson Method

Figure 2: Convergence (X axis- Number of iterations, Y axis- Logarithmic Error