Non conservative form of the momentum eq.

$$\rho \frac{\partial \bar{u}}{\partial t} + \rho(\bar{u}\cdot\nabla)\bar{u} = \rho\bar{b} + \nabla\cdot\underline{\sigma} \quad (1)$$

Where:

$$\frac{\partial \rho\bar{u}}{\partial t} = \rho\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \rho}{\partial t} \Rightarrow \rho\frac{\partial \bar{u}}{\partial t} = \frac{\partial \rho\bar{u}}{\partial t} - \bar{u}\frac{\partial \rho}{\partial t}$$

Substitute into $\circledast$

$$\frac{\partial \rho\bar{u}}{\partial t} - \bar{u}\frac{\partial \rho}{\partial t} + \rho(\bar{u}\cdot\nabla)\bar{u} = \rho\bar{b} + \nabla\cdot\underline{\sigma} \quad (2)$$

Where:

$$\nabla\cdot(\rho\bar{u}\otimes\bar{u}) = \bar{u}\,\nabla\cdot(\rho\bar{u}) + \rho(\bar{u}\cdot\nabla)\bar{u}$$

$$\rho(\bar{u}\cdot\nabla)\bar{u} = \nabla\cdot(\rho\bar{u}\otimes\bar{u}) - \bar{u}\,\nabla\cdot(\rho\bar{u})$$

Substitut in (2)

$$\frac{\partial \rho\bar{u}}{\partial t} - \bar{u}\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\bar{u}\otimes\bar{u}) - \bar{u}\,\nabla\cdot(\rho\bar{u}) = \rho\bar{b} + \nabla\cdot\underline{\sigma}$$

mass cons.

$$\frac{\partial \rho\bar{u}}{\partial t} = \rho\bar{b} + \bar{u}\cancelto{0}{\left(\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\bar{u})\right)} + \nabla\cdot(\underline{\sigma} - \rho\bar{u}\otimes\bar{u})$$

Yields:

$$\frac{\partial(\rho\bar{u})}{\partial t} = \rho\bar{b} + \nabla\cdot(\underline{\sigma} - \rho\bar{u}\otimes\bar{u}) \Rightarrow \text{Conservative form.}$$