1) Describe the strong form of the problem in the reduced domain (left half). Indicate accurately the boundary conditions in every edge.

$$\nabla \cdot \sigma + \rho b = 0$$

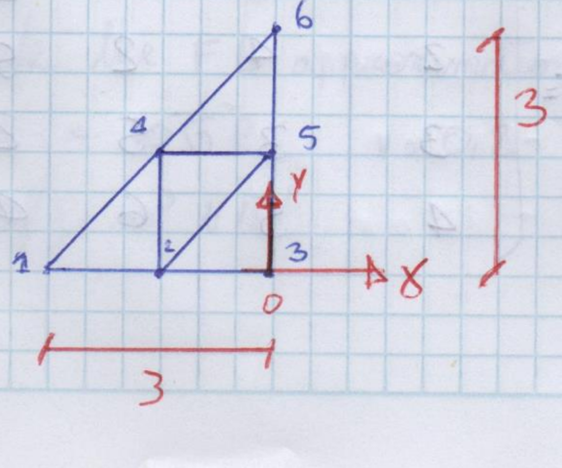
$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \rho b_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \rho b_y = 0$$

$$\sigma = D \varepsilon$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = E \begin{bmatrix} 1/(1-\nu^2) & \nu/(1-\nu^2) & 0 \\ \nu/(1-\nu^2) & 1/(1-\nu^2) & 0 \\ 0 & 0 & 1/2(1+\nu) \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$



Node	u	v
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	0	$-\delta$

2) Describe the mesh shown in figure 2 by giving arrays of nodal coordinates X and the connectivity matrix T. In order to simplify the computations select the local numbering of the nodes such that in every element, the node in the right angle vertex has local number equal to 1

Node	X	Y
1	-3	0
2	-1.5	0
3	0	0
4	-1.5	1.5
5	0	1.5
6	0	3

T =

Element	Connection (Node)		
1	2	4	1
2	4	2	5
3	3	5	2
4	5	6	4

3) Set up the linear system of equations corresponding to the discretization in fig 2
How many degrees of freedom has the system to be solved?

$$K a = f$$

$$\rightarrow \quad K_{ij}^{(e)} = \frac{t}{4A^{(e)}} \begin{bmatrix} b_i b_j d_{11} + c_i c_j d_{33} & b_i c_j d_{12} + b_j c_i d_{33} \\ c_i b_j d_{21} + b_i c_j d_{33} & b_i b_j d_{33} + c_i c_j d_{22} \end{bmatrix}$$

$$A = \frac{1}{2} \, 1{,}5 \times 1{,}5 = 1{,}125 m^2$$

$$a_i = x_j y_k - x_k y_i \; ; \; b_i = y_j - y_k \; ; \; c_i = x_k - x_j$$

$$K = \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

$$\begin{bmatrix} K_{33}^{(1)} & K_{31}^{(1)} & 0 & K_{32}^{(1)} & 0 & 0 \\ K_{13}^{(1)} & K_{11}^{(1)} + K_{22}^{(2)} + K_{33}^{(3)} & K_{31}^{(3)} & K_{12}^{(1)} + K_{21}^{(2)} & K_{23}^{(1)} + K_{31}^{(3)} & 0 \\ 0 & K_{13}^{(3)} & K_{11}^{(3)} & 0 & K_{12}^{(3)} & 0 \\ K_{23}^{(1)} & K_{21}^{(1)} + K_{12}^{(2)} & 0 & K_{22}^{(1)} + K_{11}^{(2)} + K_{33}^{(4)} & K_{13}^{(2)} + K_{31}^{(4)} & K_{32}^{(4)} \\ 0 & K_{32}^{(2)} + K_{23}^{(3)} & K_{21}^{(3)} & K_{31}^{(2)} + K_{13}^{(4)} & K_{33}^{(2)} + K_{22}^{(3)} + K_{11}^{(4)} & K_{12}^{(4)} \\ 0 & 0 & 0 & K & K_{21}^{(4)} & K_{22}^{(4)} \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{Bmatrix} r_1 + f_3^{(1)} \\ r_2 + f_1^{(1)} + f_2^{(2)} + f_3^{(3)} \\ r_3 + f_3 \\ f_2^{(1)} + f_1^{(2)} + f_3^{(4)} \\ f_{(3)}^{2} + f_2^{(3)} + f_1^{(4)} + r_5 \\ f_2^{(4)} + r_6 \end{Bmatrix}$$

4) Compute the FE approximation u^h
Use $E = 10 GPa$ $\nu = 0{,}2$ $t = 10^{-2}$ m
and $\rho g = 10^3 \, N/m^2$

$$D = E\begin{bmatrix} 1/(1-\nu^2) & \nu/1-\nu^2 & 0 \\ \nu/(1-\nu^2) & 1/1-\nu^2 & 0 \\ 0 & 0 & 1/2(1+\nu) \end{bmatrix} = \begin{bmatrix} 10416666688 & 2083333333 & 0 \\ 2083333333 & 10416666688 & 0 \\ 0 & 0 & 416666667 \end{bmatrix}$$

$$K^{(1)} = \begin{bmatrix} 7291666667 & -3125000000 & -2083333333 & 1041666667 & -5208333333 & 2083333333 \\ -3125000000 & 7291666667 & 2083333333 & -5208333333 & 1041666667 & -2083333333 \\ -2083333333 & 2083333333 & 2083333333 & 0 & 0 & -2083333333 \\ 1041666667 & -5208333333 & 0 & 5208333333 & -1041666667 & 0 \\ -5208333333 & 1041666667 & 0 & -1041666667 & 5208333333 & 0 \\ 2083333333 & -2083333333 & -2083333333 & 0 & 0 & 2083333333 \end{bmatrix}$$

$a_1 = a_2 = a_3 = 0$

$$\begin{bmatrix} K_{22}^{(1)} + K_{11}^{(2)} + K_{33}^{(4)} & K_{31}^{(4)} + K_{13}^{(2)} & K_{32}^{(4)} \\ K_{13}^{(4)} + K_{31}^{(2)} & K_{33}^{(2)} + K_{22}^{(3)} + K_{11}^{(4)} & K_{12}^{(4)} \\ K_{23}^{(4)} & K_{21}^{(4)} & K_{22}^{(4)} \end{bmatrix} \begin{Bmatrix} a_4 \\ a_5 \\ a_6 \end{Bmatrix} = \begin{Bmatrix} f_2^{(1)} + f_2^{(1)} + f_3^{(1)} \\ f_2^{(2)} + f_2^{(3)} + f_3^{(2)} + r_5 \\ f_2 + r_6 \end{Bmatrix}$$

$$\begin{bmatrix} 14{,}58 & -3{,}12 & 3{,}12 \\ -3{,}12 & 14{,}58 & -4{,}17 \\ 3{,}12 & -4{,}17 & 14{,}58 \end{bmatrix} \times 10^9 \ \text{N/m} \begin{Bmatrix} U_4 \\ V_4 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} -1{,}04 \\ 0{,}0001125 \\ 5{,}21 \end{Bmatrix} \times 10^7 \ \text{N}$$

$$\begin{Bmatrix} U_4 \\ V_4 \\ V_5 \end{Bmatrix} = \begin{Bmatrix} -0{,}128 \text{ mm} \\ -1{,}133 \text{ mm} \\ -3{,}868 \text{ mm} \end{Bmatrix}$$